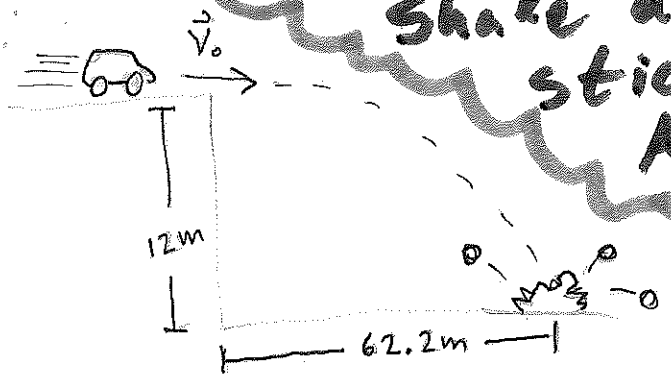
More Kin. than You Can Shake a Stick At

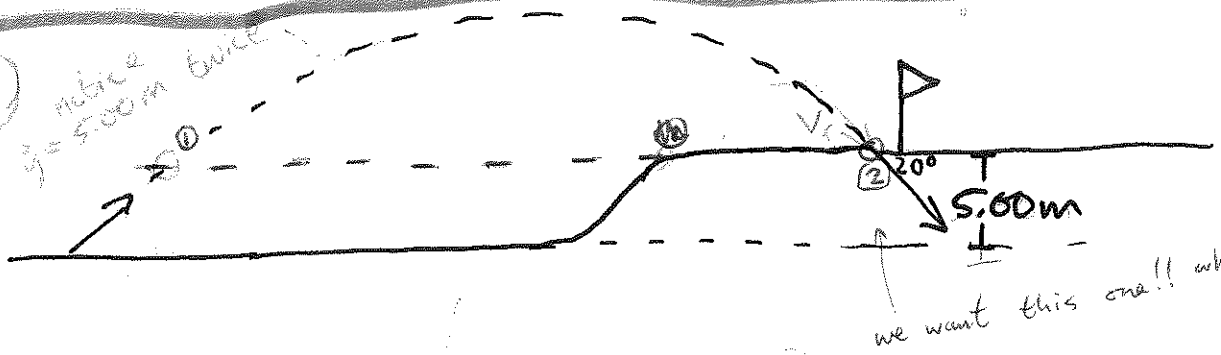
1.



$\hat{y}$	$\hat{x}$
$\vec{v}_{oy} = 0\ m/s$	$\vec{x} = 62.2\ m$
$\vec{a}_y = -9.80\ m/s^2$	$\vec{a}_x = 0\ m/s^2$
$\vec{y} = -12\ m$	$t = 1.56\ldots\ s$
$t = ?$	$\vec{v}_x = \frac{\vec{x}}{t}$
$\vec{y} = \vec{v}_{oy} t + \frac{1}{2}\vec{a}_y t^2$	$v_x = \frac{62.2\ m}{1.56492\ldots\ s}$
$t = \sqrt{\frac{2\vec{y}}{\vec{a}_y}} = 1.56\ldots\ s$	$v_x = 39.7464\ldots\ m/s$

$$\vec{v}_0 = 39.7464\ldots\ m/s \times \frac{1km}{1000m} \times \frac{3600s}{h} \approx 140\ km/h$$

2.



$\hat{y}$: $\vec{v}_y = -v\sin\theta = -19.6\sin 20 = -6.703\ldots\ m/s$

$\vec{a}_y = -9.80\ m/s^2$

$\vec{y} = +5.00\ m$

$\vec{v}_{oy} = ?$

need $\vec{v}_0$ to use any formula!

$\vec{v}_y^2 = \vec{v}_{oy}^2 + 2\vec{a}_y\vec{y}$

$\vec{v}_{oy} = \pm\sqrt{\vec{v}_y^2 - 2\vec{a}_y\vec{y}}$

$\vec{v}_{oy} = +11.955625\ldots\ m/s$ for c!

(keep for part c.)

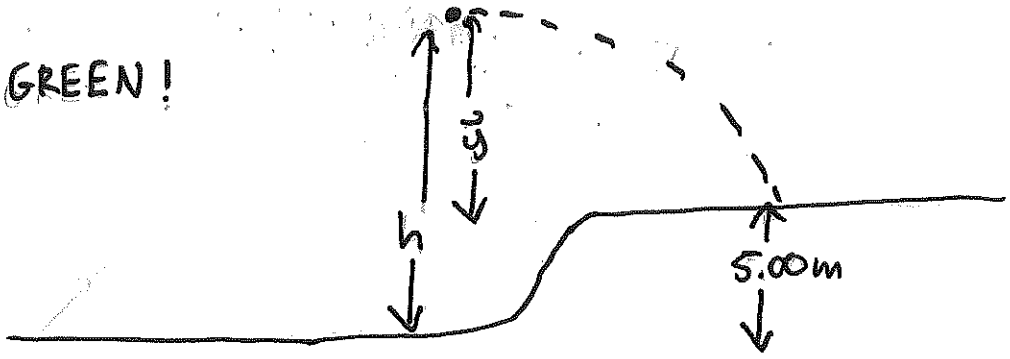
$\vec{v}_y = \vec{v}_{oy} + \vec{a}_y t$

$\Rightarrow t = \frac{\vec{v}_y - \vec{v}_{oy}}{\vec{a}_y} = \frac{-6.703\ldots - 11.95\ldots\ m/s}{-9.80\ m/s^2} = 1.904007203\ s$

b) $\hat{x}$: $\vec{v}_x = v\cos\theta = 19.6\cos 20^\circ$

$\vec{x} = \vec{v}_x t = 19.6\cos 20\,(1.904007203\ldots) = 35.1\ m$

(b) from peak to GREEN!



$\vec{y}$: $\vec{v}_{0y} = 0\ m/s$
$\vec{a}_y = -9.80\ m/s^2$
$\vec{v}_y = -19.6\sin 20\ m/s$

$\vec{v}_y^2 = \vec{v}_{0y}^{\,2} + 2\vec{a}\vec{y}$ ($\vec{v}_{0y}^{\,2} \to 0$)

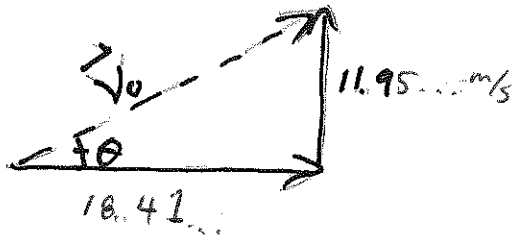
$\vec{y} = \frac{v_y^2}{2\vec{a}} = 2.29276445\ m$

$\max h = \vec{y} + 5.00\ m$

$h = 7.29\ m$

* Could also solve this using first half of motion *

(c) from a: $\vec{v}_{0y} = 11.95\ldots\ m/s$ $\vec{v}_x = 19.6\cos 20° = 18.4179\ldots$

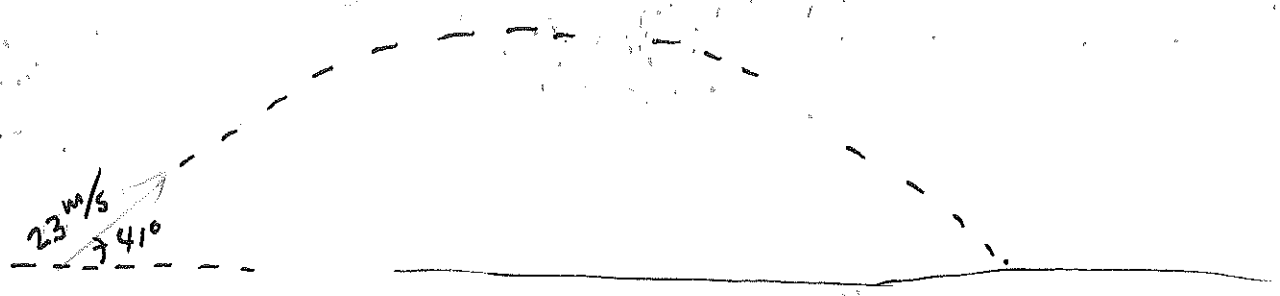


$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = 22.0\ m/s$

$\theta = \tan^{-1}\left(\frac{\vec{v}_{0y}}{v_x}\right) = 33.0°$

$\vec{v}_0 = 22.0\ m/s$ [33.0° above horizontal]

3.



$\hat{y}$: $\vec{v}_{oy} = (23 \sin 41°)\ m/s$

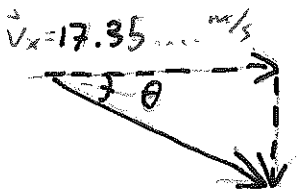
$\vec{a}_y = -9.80\ m/s^2$

$t = 2.6\ s$

$\hat{x}$: $\vec{v}_x = 23 \cos 41\ m/s$

$\vec{v}_y = \vec{v}_{oy} + \vec{a}_y t = 23 \sin 41° + (-9.80\ m/s^2)(2.6\ s) = -10.39064233\ m/s$

$\vec{v} = \vec{v}_x + \vec{v}_y$

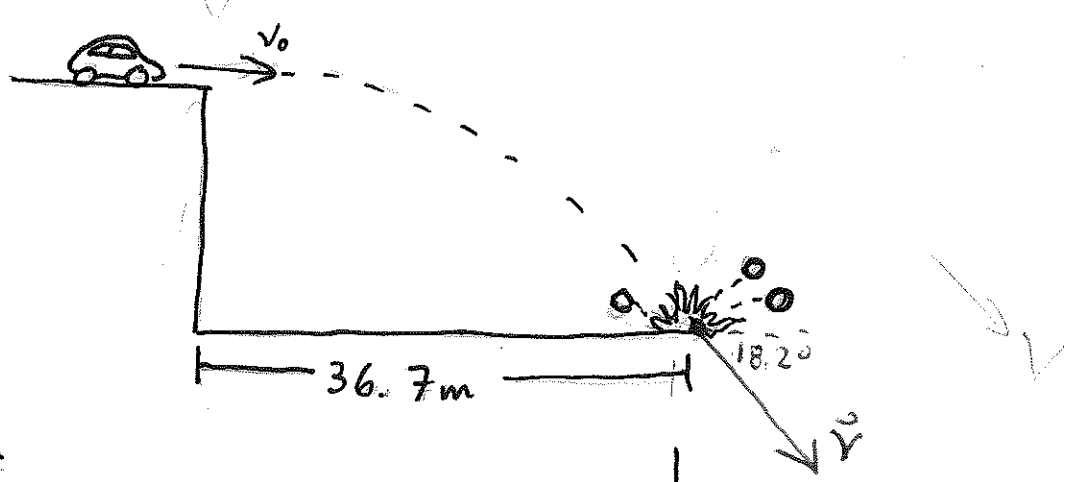


$v = \sqrt{v_x^2 + v_y^2}$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

$v = 2.0 \times 10\ m/s$ [31° below horizontal]

*4



$\hat{x}$

$v_x = v \cos 18.2°$

$\vec{x} = 36.7\ m$

$t = ?$

$\hat{y}$

$\vec{v}_{oy} = 0\ m/s$

$\vec{a}_y = -9.80\ m/s^2$

$\vec{v}_y = -\vec{v} \sin 18.2°$

$\vec{y} = ?$

$t = ?$

NOT ENOUGH INFO. TO SOLVE EITHER DIRECTLY! MUST USE $\hat{x}$ and $\hat{y}$ together!!!

$\hat{x}$: $\vec{d}_x = \vec{v}_x t$

$d_x = v\cos 18.2^\circ\, t$

$36.7 = v\cos 18.2^\circ t$ ①

$\hat{y}$: $\vec{v}_y = \vec{v}_{oy} + \vec{a}t$ ($\vec{v}_{oy} = 0$)

$-v\sin 18.2^\circ = -9.80\,m/s^2\, t$ ②

} 2 eq'ns 2 unknowns!

Let's solve in a couple of ways: A) solve ① and ② for v or t and set the two equal:

A) ① $v = \dfrac{36.7}{t\cos 18.2^\circ}$ ② $v = \dfrac{-9.80\,m/s^2\, t}{-\sin 18.2^\circ}$

$$\Rightarrow v = \frac{36.7\,m}{t\cos 18.2^\circ} = \frac{9.8\,m/s^2\, t}{\sin 18.2^\circ} = v$$

$$\Rightarrow 36.7m(\sin 18.2^\circ) = t^2(9.8\,m/s^2)\cos 18.2^\circ$$

$$t = \sqrt{\frac{36.7\,m(\sin 18.2^\circ)}{9.8\,m/s^2(\cos 18.2^\circ)}} = 1.10962\ldots$$

OR

B) solve ① or ② for v or t and substitute

① $v = \dfrac{36.7m}{t\cos 18.2^\circ}$ ⟶ ② $v\sin 18.2^\circ = 9.80\,m/s^2\, t$

$$\left(\frac{36.7m}{t\cos 18^\circ}\right)\sin 18.2^\circ = 9.8t$$

$$\Rightarrow t^2 = \frac{36.7m\sin 18.2^\circ}{9.8\,m/s^2\cos 18.2^\circ}$$

$$t = 1.10962\ldots s$$

OR c) find the quotient ② ÷ ①

② $v \sin 18.2° = 9.80 \text{ m/s}^2 \, t$ ① $v \cos 18.2 \, t = 36.7 \text{ m}$

$$\frac{v \sin 18.2°}{v \cos 18.2 \, t} = \frac{9.80 \, t}{36.7}$$

$$\tan 18.2° = \frac{9.80 \text{ m/s}^2 \, t^2}{36.7 \text{ m}}$$

$$t = \sqrt{\frac{36.7 \text{ m} (\tan 18.2°)}{9.80 \text{ m/s}^2}}$$

$t = 1.10962\ldots \text{ s}$

All give the same t. Pick your favorite!

Now use t to solve

$\hat{y}$: $v_{oy} = 0$
$a_y = -9.80 \text{ m/s}^2$
$t = 1.10962\ldots$
$y = v_{oy} t + \frac{1}{2} a_y t^2$

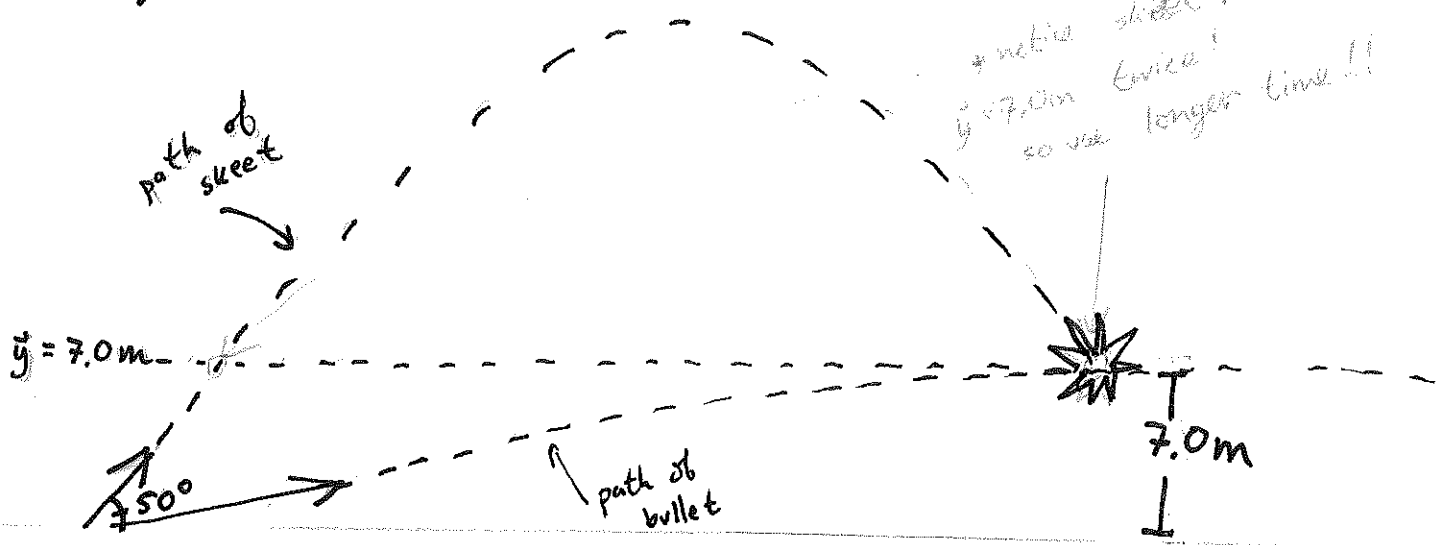
$y = \frac{1}{2} (-9.80 \text{ m/s}^2)(1.10962\ldots \text{s})^2$

$y = -6.03 \text{ m}$

The cliff is 6.03 m high

5) Two Projectiles: A skeet and a bullet!

⇒ Two 2-D problems! HOORAY!



★ Time for skeet (t_s) ≠ time for bullet(t_b): $t_s \neq t_b$

$t_s = t_b + 6.57s$

skeet

$\hat{y}$: $\vec{v}_{oy} = \vec{v}\sin\theta = 36.770...$

$\vec{y} = +7.00\,m$

$\vec{a}_y = -9.80\,m/s^2$

$t_s = ?$

$\vec{y} = \vec{v}_{oy} t_s + \frac{1}{2}\vec{a}_y t_s^2$ (quadratic)

$t_s = \dfrac{-\vec{v}_{oy} \pm \sqrt{\vec{v}_{oy}^{\,2} + 2\vec{a}_y(\vec{y})}}{\vec{a}_y}$

$t_s = 7.308645607s$

$\hat{x}$: $v_x = v\cos\theta = 30.853805\frac{26}{}\,m/s$

$t_s = 7.308645607s$

$\vec{x} = \vec{v}_x t_s = 225.50m$

bullet

$\hat{x}$: $t_b = t_s - 6.57s = 0.738645607s$

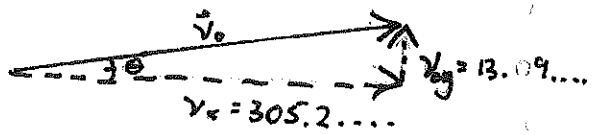
$\vec{x} = 225.499....m$

$v_x = \dfrac{\vec{x}}{t_x} = 305.2878487\,m/s\ (\vec{v}_x)$

$\hat{y}$: $t_b = 0.738645607s$, $\vec{y} = +7.00m$, $\vec{a}_y = -9.80\,m/s^2$

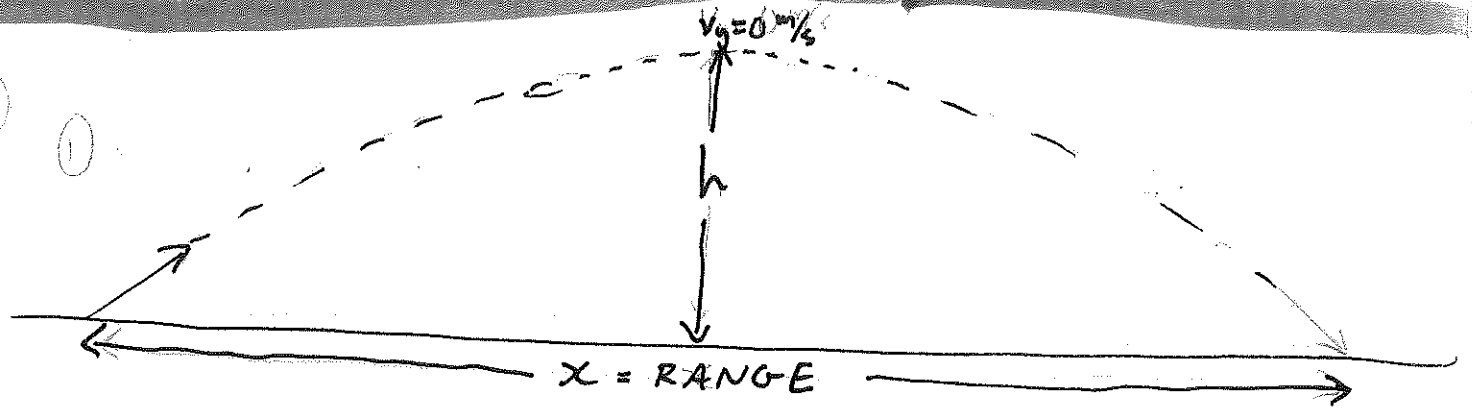
$\vec{y} = \vec{v}_{oy} t + \frac{1}{2}\vec{a}_y t^2$

$\vec{v}_{oy} = \dfrac{\vec{y} - \frac{1}{2}\vec{a}_y t^2}{t} = 13.09616796\,m/s\ (\vec{v}_{oy})$



$\vec{v}_{0b} = 306\ m/s$ [2.46° above horizontal]

6.



$\hat{x}$:

$v_x = \vec{v}_0 \cos\theta$

$\vec{x} = 98\ m$

$t = ?$

$\vec{x} = \vec{v}_x t$

$\vec{v}_x = \frac{\vec{x}}{t} = \frac{98\ m}{4.33307...\ s} = 22.6167...$

$\hat{y}$: (from start to highest point)

$\vec{v}_{oy} = \vec{v}_0 \sin\theta$

$\vec{a}_y = -9.80\ m/s^2$

$\vec{y} = 23\ m$

$\vec{v}_y = 0\ m/s$

$\vec{v}_y^{\,2} = \vec{v}_{oy}^{\,2} - 2\vec{a}\vec{d}$

$\vec{v}_{oy} = +\sqrt{-2(-9.8\ m/s^2)(23\ m)}$

$\vec{v}_{oy} = 21.23205124$

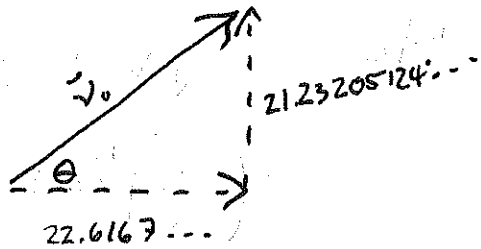
$\vec{v}_y = \vec{v}_{oy} + \vec{a}_y t$

$t = \frac{\vec{v}_y - \vec{v}_{oy}}{a_y} = \frac{0 - 21.23205124\ m/s}{-9.80\ m/s^2}$

$t_{up} = 2.166535841\ s$

$t = 2t_{up} = 4.333071682\ s$

use to solve $\hat{x}$



$\vec{v}_0 = 31\ m/s$ [43° above x]

More Projectiles Than You Can Shake a Stick At.

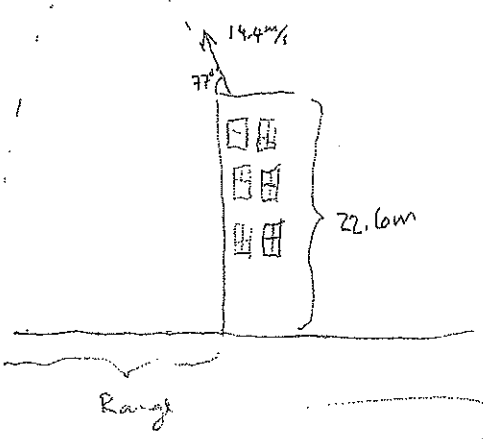
(7)

$\hat{x}$: $\vec{v}_x = 14.4\ m/s \cos 77°$

$\hat{y}$: $\vec{v}_{oy} = 14.0\ m/s \sin 77°$

$\vec{a}_y = -9.80\ m/s^2$

$\vec{d}_y = -22.6\ m$



$\hat{y}$: $\vec{d}_y = \vec{v}_{oy} t + \frac{1}{2}\vec{a}_y t^2$

$t = \dfrac{-\vec{v}_{oy} - \sqrt{\vec{v}_{oy}^2 + 2\vec{a}_y \vec{d}_y}}{\vec{a}}$ ← quadratic formula

$t = 4.012829603\ s$

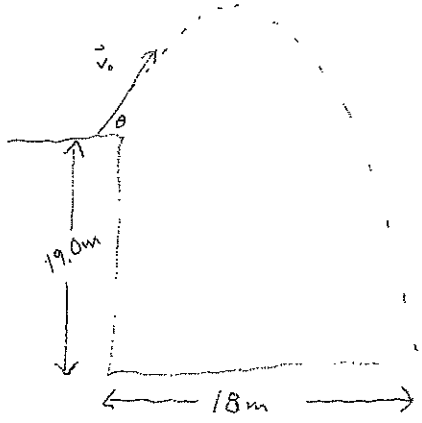
$\hat{x}$: $d_x = v_x t$

$d_x = 14.0\ m/s \cos 77° (4.012829603_s)$

$d_x = 12.9987396\ m$

$R = 13\ m$

(8)



$\hat{x}$: $d_x = 18m$

$t = 2.7s$

$v_x = \dfrac{d_x}{t} = \dfrac{18m}{2.7s}$

$v_x = 6.\overline{6}\ m/s$

$\hat{y}$: $\vec{a}_y = -9.80\ m/s^2$

$\vec{d}_y = -19.0\ m$

$t = 2.7\ s$

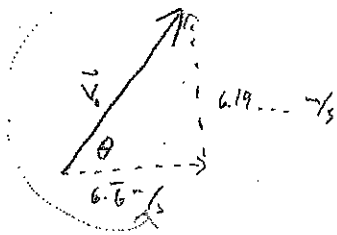
$\vec{v}_{oy} = ?$

$\vec{d}_y = \vec{v}_{oy} t + \frac{1}{2}\vec{a}_y t^2$

$\vec{v}_{oy} t = \vec{d}_y - \frac{1}{2}\vec{a}_y t^2$

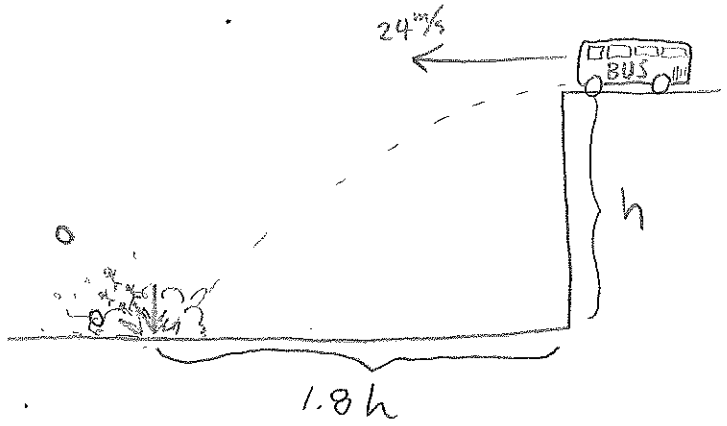
$\vec{v}_{oy} = \dfrac{\vec{d}_y - \frac{1}{2}\vec{a}_y t^2}{t}$

$\vec{v}_{oy} = 6.192962963$



$\vec{v}_0 = 9.1\ m/s$ [43° above horizontal]

10.



$\hat{x}$: $\vec{v} = 24\ m/s$, $\vec{d}_x = 1.8h$, $t = ?$

$\vec{d}_x = \vec{v}_x t$

$1.8h = 24t$ ①

$\hat{y}$: $\vec{v}_{oy} = 0$, $\vec{a}_y = -9.80\ m/s^2$, $\vec{d}_y = -h$, $t = ?$

$\vec{d}_y = \cancel{\vec{v}_{oy} t}^{\,0} + \frac{1}{2}\vec{a}_y t^2$

$-h = -4.9t^2$

$h = 4.9t^2$ ②

Substitute ② → ①

$1.8(4.9t^2) = 24t$

$t = \frac{24}{1.8(4.9)} = 2.721088435\ s$

$h = 4.9t^2 = 36.28117914\ m$

$\boxed{h = 36m}$