AP1 - Rotational Dynamics

1a. $I = \sum mr^2 = 2ML^2 + 3M(2L)^2 = 14ML^2$

b. $I = \sum mr^2 = ML^2 + 3ML^2 = 4ML^2$

c. $I = \sum mr^2 = 2ML^2 + M(2L)^2 = 6ML^2$

d. $\sum \tau = I\alpha \qquad F(2L) = 14ML^2\alpha$

$\alpha = \dfrac{F}{7ML}$

e. $\sum \tau = I\alpha \qquad F(L) = 4ML^2\alpha$

$\alpha = \dfrac{F}{4ML}$

f. $\sum \tau = I\alpha \qquad F(0) = 6ML^2\alpha$

$\alpha = 0$

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2. $I = \sum mr^2 = 2ML^2 + 3M(2.5L)^2$

$I = 21ML^2 \qquad (20.75ML^2)$

b. $I = \sum mr^2 = M(1.5L)^2 + 2M(0.5L)^2 + 3ML^2$

$I = 5.8ML^2 \qquad (5.75ML^2)$

c. $I = \sum mr^2 = 3M(0.5L)^2 + 2M(2L)^2 + M(3L)^2$

$I = 18ML^2 \qquad (17.75ML^2)$

d. $\vec{\alpha} = \dfrac{\sum \vec{\tau}}{I} = \dfrac{2F(L) - F(3L)}{20.75ML^2} = \dfrac{-FL}{20.75ML^2} = 0.048\dfrac{F}{ML}$ ccw.

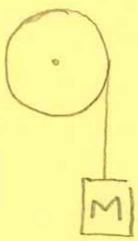
e. $\vec{\alpha} = \dfrac{\sum \vec{\tau}}{I} = \dfrac{2F\cos\theta(1.5L) - 2F(0.5L) - F(1.5L)}{5.75ML^2} = \dfrac{F\cancel{L}(3\cos\theta - 1 - 1.5)}{5.75ML^{\cancel{2}}} = \dfrac{F(3\cos\theta - 2.5)}{5.75ML}$ ccw

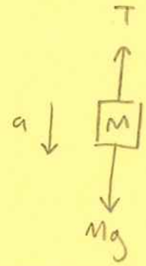
f. $\vec{\alpha} = \dfrac{\sum \vec{\tau}}{I} = \dfrac{2F\cos\theta(3L) - 2F(2L)}{17.75ML^2} = \dfrac{FL(6\cos\theta - 4)}{17.75ML^2} = \dfrac{2F(3\cos\theta - 2)}{17.75ML} = \dfrac{0.11F(3\cos\theta - 2)}{ML}$

3. $\vec{\alpha} = \dfrac{\Sigma \vec{\tau}}{I} = \dfrac{100N(0.60m) - 200N(0.42m) - 250N(0.28m)}{50 kgm^2} = -1.88 \, rad/s^2$

$\vec{\alpha} = 1.88 \, rad/s^2$ CCW

4.





$\Sigma \vec{F} = m\vec{a}$

$Mg - T = Ma$

$T = Mg - Ma$

$\Sigma \vec{\tau} = I\alpha$

$TR = \frac{1}{2} m R^2 \alpha$

$TR = \frac{1}{2} m R^2 \left(\frac{a}{R}\right)$

$T = \frac{1}{2} ma$

$\frac{1}{2} ma = Mg - Ma$

$\Rightarrow \frac{1}{2} ma + Ma = Mg$

$\rightarrow a = \dfrac{Mg}{(\frac{1}{2}m + M)} = 1.351724138 \, m/s^2$

a. $\vec{\alpha} = \frac{a}{r} = 2.7 \, rad/s^2$ cw

b. $\vec{a} = 1.4 \, m/s^2$ down

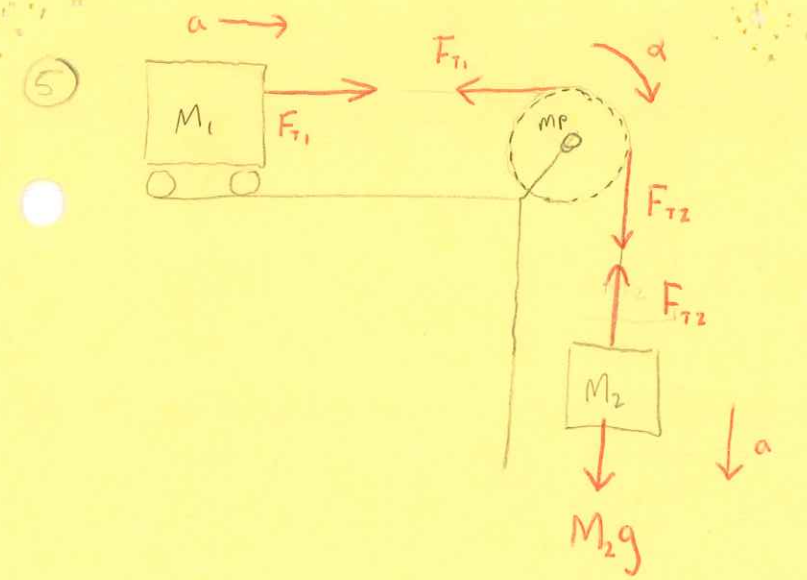
c. $T = \frac{1}{2} ma = 84N$

d. $\vec{\omega}^2 = \cancel{\vec{\omega}_0^2}^{\,0} + 2\vec{\alpha}\vec{\theta}$

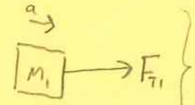
$\omega^2 = 2\alpha\left(\frac{d}{r}\right)$

$\rightarrow \omega = \sqrt{2(\alpha)\left(\frac{d}{r}\right)} = 3.3 \, rad/s$

(5)

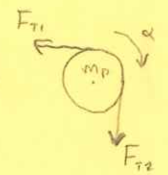


M_1:



$F_{T1} = M_1 a$ (1)

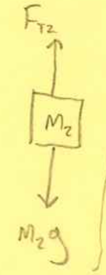
M_p



$F_{T2} r - F_{T_1} r = I\alpha$

$F_{T2} \cancel{r} - F_{T_1} \cancel{r} = \frac{1}{2} m_p \cancel{r^2} \left(\frac{a}{\cancel{r}}\right)$ (2)

M_2



$M_2 g - F_{T2} = M_2 a$

$F_{T2} = M_2 g - M_2 a$ (3)

sub (1) and (3) INTO (2) $\left[M_2 g - M_2 a\right] - M_1 a = \frac{1}{2} m_p a$

$$M_2 g = \frac{1}{2} m_p a + M_1 a + M_2 a = \left(\frac{1}{2} m_p + M_1 + M_2\right) a$$

$$a = \frac{M_2 g}{\left(\frac{1}{2} m_p + M_1 + M_2\right)} = 3.015384615 \ m/s^2$$

$\boxed{\begin{array}{l} \vec{a} = 3.0 \, m/s^2 \text{ right for } M_1 \\ F_{T_1} = M_1 a = 6.0 N \\ F_{T2} = M_2 g - M_2 a = 6.8 N \end{array}}$

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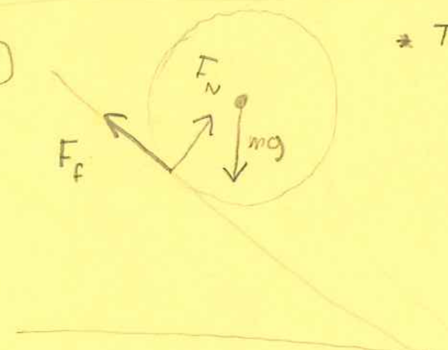
(6)



* Treat the wheel as a ring so $I = mr^2$

$\sum \vec{F} = m\vec{a}$

$mg\sin\theta - F_f = ma$ (1)

$\sum \vec{\tau} = I\vec{\alpha}$

$F_f r = mr^2 \left(\frac{a}{r}\right)$

$F_f = ma$ (2)

$$mg\sin\theta - ma = ma$$

$$g\sin\theta = 2a$$

$$a = \frac{g\sin\theta}{2} = 1.5 \ m/s^2$$

b. $F_{f\max} = \mu mg\cos\theta$ AND $a = \frac{g\sin\theta}{2}$, so θ max is when F_f is max . . .

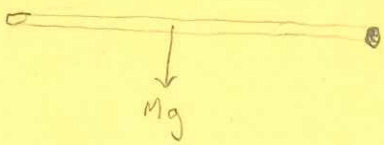
So put $F_{f\max}$ and a into (2) to get $\mu \cancel{mg} \cos\theta = \cancel{m}\left(\frac{\cancel{g}\sin\theta}{2}\right)$

$$2\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}(2\mu) = \tan^{-1}\left(2(0.6)\right) = 50.194\ldots^\circ$$

$$\theta = (5.0 \times 10)^\circ$$

7



$$\sum \vec{\tau} = I\vec{\alpha}$$

$$\cancel{M}g\left(\frac{L}{2}\right) = \frac{1}{3}\cancel{M}L^2\alpha$$

$$\frac{3}{2}\frac{g}{L} = \alpha$$

$$\vec{\alpha} = 7.35 \text{ rad/s}^2 \text{ out}$$

b. $a = r\alpha = 14.7 \text{ m/s}^2$ down