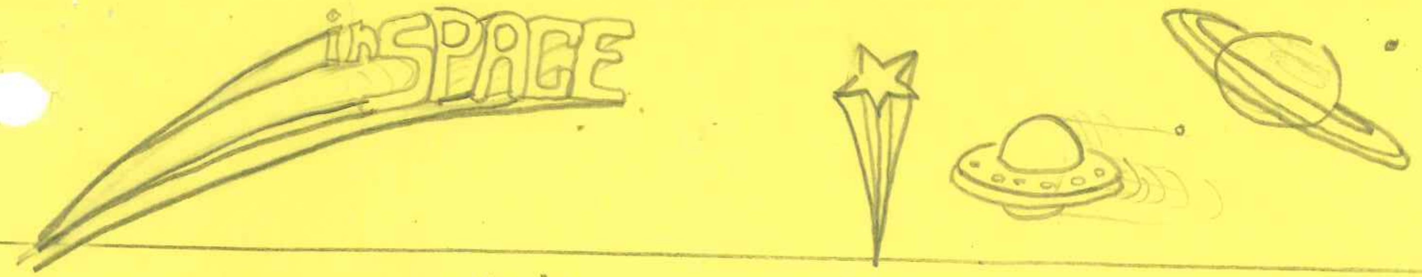
Gravitational Potential Energy



1a) $GPE = -\frac{GMm}{r} = \frac{-6.67\times10^{-11}\,\frac{Nm^2}{kg^2}(5.98\times10^{24}\,kg)(7500\,kg)}{(6.38\times10^{6}\,m + 3\times10^{6}\,m)} = -3.2\times10^{11}\,J$

b) $W_{nc} = \Delta E_k + \Delta E_p$; $\Delta E_k = 0\,J$ (minimum energy means $\vec{v}_i = \vec{v}_f = 0\,J$!)

$W_{nc} = \Delta E_p = \cancel{E_{pf}}^{0} - E_{pi} = -E_{pi} = 3.2\times10^{11}\,J$

2a) $W_{nc} = \cancel{\Delta E_k}^{0} + \Delta E_p = E_{pf} - E_{pi} = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right)$

$= -GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = -GMm\left(\frac{1}{r_e + 2.6\times10^{6}\,m} - \frac{1}{r_e}\right)$

$W_{nc} = 1.0\times10^{11}\,J$

b) $W_{nc} = \cancel{\Delta E_k}^{0} + \Delta E_p = \cancel{E_{pf}}^{0} - E_{pi} = -\left(\frac{-GMm}{r_e + 2.6\times10^{6}}\right) = 2.6\times10^{11}\,J$

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3) $E_{rg} = -\frac{GMm}{r} = \frac{-6.67\times10^{-11}\,\frac{Nm^2}{kg^2}(7.35\times10^{22}\,kg)(250\,kg)}{1.74\times10^{6}\,m} = -7.0\times10^{8}\,J$

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4) $W_{nc} = 0\,J$

$E_{ki} + E_{pi} = \cancel{E_{kf}}^{0} + \cancel{E_{pf}}^{0}$

$\frac{1}{2}\cancel{m}V_i^2 + \frac{-GM\cancel{m}}{r} = 0\,J$

$V_{esc} = \sqrt{\frac{2GM}{r}}$

a) $V_{esc} = \sqrt{\frac{2G(7.35\times10^{22}\,kg)}{1.74\times10^{6}\,m}} =$ a) $\underline{2370\,m/s}$

b) $V_{esc} = \sqrt{\frac{2G(6.42\times10^{23}\,kg)}{3.40\times10^{6}\,m}} =$ b) $\underline{5020\,m/s}$

c) $V_{esc} = \sqrt{\frac{2G(4.87\times10^{24}\,kg)}{6.05\times10^{6}\,m}} =$ c) $\underline{10\,400\,m/s}$

d) $V_{esc} = \sqrt{\frac{2G(1.99\times10^{30}\,kg)}{6.96\times10^{8}\,m}} =$ d) $\underline{618\,000\,m/s}$

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5)

$W_{nc} = 0\,J$

$E_i = E_f$

$\frac{1}{2}\cancel{m}V_i^2 + \left(\frac{-GM\cancel{m}}{r_e}\right) = \frac{-GM\cancel{m}}{r_e + 2.5\times10^{6}}$

$V_i = \sqrt{2(-GM)\left(\frac{1}{r_e + 2.5\times10^{6}} - \frac{1}{r_e}\right)}$

$V_i = 5900\,m/s$

6) $\frac{1}{2}g = \frac{1}{2}(9.80 m/s^2) = 4.9 m/s^2$

on earths surface

$$g = \frac{GM}{r^2}$$

$$r = \sqrt{\frac{GM}{g}} = 9.02226 \times 10^6 m$$

$$h = r - r_e = 2.6 \times 10^6 m$$

6 a. $0J$ (F_g is $\perp$ to motion!)

b. $E_g = \frac{-GMm}{r}$; G, M, m all are the same for both satellites

so... $E_g \propto \frac{1}{r}$

so... $\frac{1}{2}r \Rightarrow 2E_{pg}$

$E_{pg_2} = -13.8 \times 10^{12} J$

$= -1.4 \times 10^{13} J$

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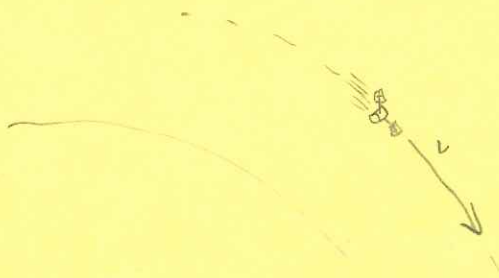
7 a. $\mathcal{E}_{pg} = \dfrac{-GMm}{r_e + h} \Rightarrow r_e + h = \dfrac{-GMm}{\mathcal{E}_{pg}} = \dfrac{-(6.67\times10^{-11}\,\frac{Nm^2}{kg^2})(5.98\times10^{24}\,kg)(38700\,kg)}{-1.96\times10^{12}\,J}$

$r_e + h = 7.8755\ldots\ldots \times 10^6\,m$

$h = 1.50\times10^6\,m$

b. $\mathcal{E}_k = \frac{1}{2}mv^2$; We need to find v! $\Rightarrow$ CIRCULAR MOTION



$\Sigma\vec{F} = m\vec{a}$

$\dfrac{GMm}{r^{\cancel{2}}} = \dfrac{mv^2}{\cancel{r}}$

$\Downarrow$

$\dfrac{1}{2}\dfrac{GMm}{r} = \dfrac{1}{2}mv^2$

* [NOTICE: I want $\mathcal{E}_k$. $\mathcal{E} = \frac{1}{2}mv^2$, right now I have mv^2. Instead of solving for v I can just take $\frac{1}{2}$ of each side!]

So... $\mathcal{E}_k = \frac{1}{2}\dfrac{GMm}{r}$; AND $\mathcal{E}_p = \dfrac{-GMm}{r}$ so... $\mathcal{E}_k = -\frac{1}{2}\mathcal{E}_p$

(for a satellite in circular orbit. NOT IN GENERAL)

$\mathcal{E}_k = -\frac{1}{2}(-1.96\times10^{12}\,J) = 9.80\times10^{11}\,J$

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a. $\cancelto{0}{W_{nc}} = \Delta\mathcal{E}_k + \Delta\mathcal{E}_p = \mathcal{E}_k - \mathcal{E}_{ko} + \mathcal{E}_p - \mathcal{E}_{po}$

$0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 + \left(\dfrac{-GMm}{r_e + h}\right) - \left(\dfrac{-GMm}{r_e + h_o}\right)$

$v = \sqrt{\dfrac{2}{\cancel{m}}\left[\frac{1}{2}v_o^2 - \left(\dfrac{-GM}{r_e + h}\right) + \left(\dfrac{-GM}{r_e + h_o}\right)\right]}$

$v = \sqrt{2\left[\frac{1}{2}(18000\,m/s)^2 + (6.67\times10^{-11}\,\frac{Nm^2}{kg^2})(5.98\times10^{24}\,kg)\left[\dfrac{1}{6.38\times10^6 + 1.2\times10^5} - \dfrac{1}{6.38\times10^6 + 1.9\times10^7}\right]\right]}$

$v = 2.03788243\times10^4\,m/s$

$v = 2.0\times10^4\,m/s$ $(20\,km/s)$

b. Theres a lot here! We have heat lost $\Rightarrow W_{nc} \neq 0$; $W_{nc} = -8.95 \times 10^{17}$ J

We have a change in mass! $m_o = 1.25 \times 10^{10}$ kg

$m = 0.30(m_o) = 3.75 \times 10^{9}$ kg

$$W_{nc} = \Delta E_k + \Delta E_p$$

$$-8.95 \times 10^{17}\text{ J} = \frac{1}{2} m v^2 - \frac{1}{2} m_o v_o^2 + \left(\frac{-GMm}{r_e}\right) - \left(\frac{-GMm_o}{r_e + h_o}\right)$$

* everything but v is known, so apart from some significant algebra we're DONE!

ALGEBRA...

$$v = \sqrt{\frac{2}{m}\left(-8.95 \times 10^{17}\text{ J} + \frac{1}{2} m_o v_o^2 + GM\left(\frac{m}{r_e} - \frac{m_o}{r_e + 1.9 \times 10^{7}}\right)\right)}$$

$v = 2.4958591 6 \times 10^{4}$ m/s

$v = 2.5 \times 10^{4}$ m/s ; 25 km/s

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9. B

10. $E_{pg} \propto \frac{1}{r} \Rightarrow$ DOUBLE R, $\frac{1}{2} E_{pg}$

$E_{pgB} = \frac{1}{2} E_{pgA} = -13\,000$ J

11. 26000 J

12. 13 000 J

13. 13 000 J