Scalar Multiples of Vectors:

Recall some of the physics you learned in grade 11:

$$\sum\_{}^{}\rightharpoonaccent{F}=m\rightharpoonaccent{a}$$

$$\rightharpoonaccent{d}=\rightharpoonaccent{v}\_{o}t+ \frac{1}{2}\rightharpoonaccent{a}t^{2}$$

$$\rightharpoonaccent{v}=\rightharpoonaccent{v}\_{o}+\rightharpoonaccent{a}t$$

In each of these equations you have a vector quantity being multiplied by a scalar quantity. We have already seen that vectors behave differently than scalars under addition, so let’s examine ***scalar multiples of vectors.***

First let’s just very quickly recall what multiplication *is*.

 5z simply means 5 times z, which means z+z+z+z+z.

 12R simply means 12 times R, which means R+R+R+R+R+R+R+R+R+R+R+R.

Multiplication is just a short-hand for addition. This remains true for vectors.

If we have a vector $\rightharpoonaccent{A}$ = 22m/s [38o below –x] that vector can be sketched as:

Then 2$\rightharpoonaccent{A}$ is simply $\rightharpoonaccent{A}+\rightharpoonaccent{A}$ . That can be sketched as:

It should be clear that the result is a vector with 2 times the magnitude, but with no change in the direction.

When a vector is multiplied by a scalar, the multiplication only affects the magnitude not the direction!

Examples:

1. $\rightharpoonaccent{B}$ = 36N [19o above –x]

* Find 5$\rightharpoonaccent{B}$.

We need only multiply the magnitude whilst maintaining direction. So…

 5$\rightharpoonaccent{B}$=180N [19o above –x]

2. $\rightharpoonaccent{E}$ = 2.6x105N/C [66o above +x]

* Find 0.50$\rightharpoonaccent{E}$.

We need only multiply the magnitude whilst maintaining direction. So…

 0.50$\rightharpoonaccent{E}$ = 1.3x105N/C [66o above +x]

3. $\overbar{v}$ = 12m/s [27o below –x] ; t=3.0s

* Find $\rightharpoonaccent{d}$ = $\overbar{v}t$

No problem. The only difference here is that our scalar value has units. Hopefully we know how to do operations with units from Physics 11. Again just multiply the magnitude and maintain direction.

 $\rightharpoonaccent{d}$ = $vt$ = 12m/s(3.0s) = 12(3)($\frac{m}{s} s$)= 36$\frac{m}{s} s$ = 36m

 $\rightharpoonaccent{d}$ = 36m [27o below –x]

4. $\rightharpoonaccent{a}$ = 4.2m/s2 [38o below +x]; t=2.7s

* Find $\frac{1}{2}\rightharpoonaccent{a}t^{2}$

Don’t panic. $\frac{1}{2}$ and t2 are both just scalars. So we just need to multiply the magnitude of $\rightharpoonaccent{a}$ by those scalars and maintain direction. Careful with units.

 $\frac{1}{2}at^{2}$ = $\frac{1}{2}$ (4.2m/s2)(2.7s)2 = 15.309$\frac{m}{s^{2}} s^{2}$ = 15m

 $\frac{1}{2}\rightharpoonaccent{a}t^{2}$ = 15m [38o below +x]

Okay, so far, so good. There is only one more little thing to consider: Negative numbers.

First let’s just think about this conceptually. We already know difference between 5m $\hat{x}$ and -5m $\hat{x}$. We already know the know the difference between 125N $\hat{y}$ and -125 $\hat{y}$. The sign simply refers to direction.

So, in general the difference between a vector $\rightharpoonaccent{A}$ and the vector $-\rightharpoonaccent{A}$ is that they are in **opposite directions**. This makes logical sense also since we know that $\rightharpoonaccent{A}$+$\left(-\rightharpoonaccent{A}\right)=0$.

Consider a vector, $\rightharpoonaccent{K}$, which is 13m long and is directed at 55o above –x:

 $\rightharpoonaccent{K}$ = 13m [55o above –x]

 $ \rightharpoonaccent{K}$

 55o

Then $-\rightharpoonaccent{K}$ is just a vector of equal magnitude in the opposite direction:

55o

 $ -\rightharpoonaccent{K}$

$-\rightharpoonaccent{K}$ = 13m [55o below +x]

Try these:

 $\rightharpoonaccent{T}$ = 15m/s [ 42o below –x] ; $-\rightharpoonaccent{T}$ =

 $\rightharpoonaccent{E}$ = 22000N/C [12o above –x] ; $-\rightharpoonaccent{E}$ =

 $\rightharpoonaccent{B}$ = 3.1x10-5 T [78o below +x] ; $-\rightharpoonaccent{B}$ =

Okay. Good. Now just combine those concepts. If $\rightharpoonaccent{F}$ = 314N [51o below –x], Find $-3\rightharpoonaccent{F}$.

All we need to recognize is that $-3\rightharpoonaccent{F}$= $-(3\rightharpoonaccent{F})$. I’ll leave the rest to you.

Practice, Practice, Practice:

 $\rightharpoonaccent{A}$ = 27m [23o above –x] $\rightharpoonaccent{B}$ = 72m [51o above –x] $\rightharpoonaccent{C}$ = 61m [39o below +x]

1. Find 4$\rightharpoonaccent{A}$

2. Find 10$\rightharpoonaccent{B}$

3. Find -2$\rightharpoonaccent{C}$

4. Find 7$\rightharpoonaccent{A}$ - $\rightharpoonaccent{B}$

$\rightharpoonaccent{v}\_{o}$= 5.0m/s [63o N of W] $\rightharpoonaccent{a}$ = 2.0m/s2 [26o S of E] $t$ = 4.0s

5. Find $∆\rightharpoonaccent{v}= \rightharpoonaccent{a}t$

6. Find $\frac{1}{2}\rightharpoonaccent{a}t^{2}$

7. Find $\rightharpoonaccent{v}=\rightharpoonaccent{v}\_{o}+ \rightharpoonaccent{a}t$

8. Find $\rightharpoonaccent{v}\_{o}t$