The Addition of Vectors: N4

What we have learned up to this point:

* Graphically we can add vectors by drawing each successive vector beginning at the end of the previous vector. This is sometimes referred to as the “tail to head” method. The resultant vector is simply a single vector that points from the start of the first vector to end of the last vector.
* The order of addition does not change the result.
* Given a series of one dimensional (Left, Right, Up, Down) vectors, it is easiest to group them as horizontal and vertical. We can then simply add all the horizontal vectors together, then add all the vertical components together. This will ALWAYS RESULT IN A RIGHT-ANGLED TRIANGLE.
* Any changes in the horizontal direction have no effect on the vertical direction. Any changes in the vertical direction have no effect on the horizontal direction.
* Given a vector in standard form (magnitude and angle) we can convert to component. Given a vector in component form ($\hat{x}, \hat{y})$ we can convert it to standard form. These operations involve simple trigonometry and the Pythagorean theorem.

You will usually be given vector quantities in **standard form.** The reason is that this is a more natural description of the vector, in that you immediately know its magnitude and direction. There is however a disadvantage to the standard form. The disadvantage comes when we attempt to add or subtract (or multiply) vector quantities.

Consider the following vector sum:

 **d1**= 58.0m [63.5o above –x] ; **d2**= 71.5m [13.2o below +x]

Find **d3** = **d1** + **d2**

Let’s look at this carefully. 1. Draw each vector (to scale)

 2. Add the vectors tail to head in the order given.

 3. Draw the resultant vector, **d3**.

 d1=58.0m 13.2o

1.

 d2=71.5m

 63.5o

2.

13.2o

 **d2**

 **d1**

63.5o

3.

 **d2**

13.2o

 **d1**

 **d3**

63.5o

 θ

Okay. So now what? It’s a triangle, yes. But it is **NOT** a right triangle. So how can we find $\left‖d\_{3}\right‖$ ? How can we find θ?

Yes, you’re right, we could use the sine law and the cosine law. But those are very cumbersome, and if we are not very careful, they give incorrect results. Also, that wouldn’t work if we had to add three or more vectors together. Any other thoughts?

Anything?

Hmmmmmm…

YES! OF COURSE! CONVERT THE VECTORS TO COMPONENT FORM! THEN ADDITION IS SIMPLE!

 1. Draw each vector (to scale), and find the components

 2. Add the vectors in the order given, include the components.

 3. Remove the vectors and draw the resultant vector, **d3**.

 4. Rearrange the components (order of addition is not important!)

 5. Solve the resulting right triangle.

1.

 d1=58.0m **d2x**

 13.2o  **d2y**

 **d1y**

 d2=71.5m

 63.5o

 **d1x**

2. 3. **d2x**

 **d2y**

 **d1y**

 **d3**

 **d1x**

4.

 **d2y**

 **d1y d3y**

 (Can you see the right angle triangle?)

 **d1x d2x**

 **d3x**

5. **d3x** = **d1x** + **d2x** = - 25.87947316m **x** + 69.61089156m **x** = 43.7314184m **x**

 **d3y** = **d1y** + **d2y** = 51.90619297m **y** + (-16.32708721m) **y** = 35.57910576m **y**

 **d3**

d3y=35.57910576m

θ

d3x=43.7314184m

$d\_{T}= \sqrt{d\_{tx}^{2}+ d\_{ty}^{2}}$ θ = tan-1 $\frac{d\_{ty}}{d\_{tx}}$

so…. **dT**= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

And that is vector addition in 2D using components. As we get more comfortable we will drop most (if not, all) of the drawing steps and the process will become much quicker. For now, however, I thing it is important to show the steps in order to build a deep understanding of the process.

Try this example:

 $\rightharpoonaccent{F}\_{1}$= 124N [77.0o above -x] $\rightharpoonaccent{F}\_{2}$= 96.6N [19o below +x]

 Find $\sum\_{}^{}\rightharpoonaccent{F}$ = $\rightharpoonaccent{F}\_{1}$+$\rightharpoonaccent{F}\_{2}$