The Addition of Vectors:

Now adding vectors requires a bit more thought than adding scalars, but it is still quite simple in concept. Consider the following sets of instructions:

1. Go 3.0m left, then 4.0m up, then 6.0m left, then 9.0m down, then 1.0m right, then 2.0m down.

Can you tell me where you are compared to where you began?

- Horizontally I am ______________ m (Right/ Left)
- Vertically I am ______________ m (Up/ Down)

Would the order in which you made the above moves change the overall result?

2. Go 12m right, 6.0m down, 4.0m left, 3.0m up, 9.0m right, 4.0m up.

Can you tell me where you are compared to where you began?

- Horizontally I am ______________ m (Right/ Left)
- Vertically I am ______________ m (Up/ Down)

Would the order in which you made the above moves change the overall result?

3. Go 32m up, 49m left, 18m right, 26m up, 12m up, 14m left, 19m left, 16m down.

Can you tell me where you are compared to where you began?

- Horizontally I am ______________ m (Right/ Left)
- Vertically I am ______________ m (Up/ Down)

Would the order in which you made the above moves change the overall result?

4. Now what if the numbers are a bit tougher? Go 7.61cm R, 12.06cm D, 9.33cm R, 4.28cm U, 2.55cm L, 1.88cm U.

Can you tell me where you are compared to where you began?

- Horizontally I am ______________ m (Right/ Left)
- Vertically I am ______________ m (Up/ Down)

Would the order in which you made the above moves change the overall result?

If you could answer the above questions you already know how to add vectors in 2-dimensions!

- The first set of instructions could be re-written as the sum of 3 two dimensional displacement vectors.

  $\mathbf{d_1}$= -3.0m $\mathbf{x}$ + 4.0m $\mathbf{y}$; (3.0m left , 4.0m up) $\mathbf{d_2}$= -6.0m $\mathbf{x}$ + (-9.0m) $\mathbf{y}$; (6.0m left, 9.0m down) $\mathbf{d_3}$= 1.0m $\mathbf{x}$ + (-2.0m) $\mathbf{y}$ (1.0m right, 2.0m down)

  You then found $\mathbf{d_T} = \mathbf{d_1} + \mathbf{d_2} + \mathbf{d_3}$

- The second set of instructions could be re-written as the sum of 3 two dimensional displacement vectors.

  $\mathbf{d_1}$= 12m $\mathbf{x}$ + (-6.0)m $\mathbf{y}$; $\mathbf{d_2}$= -4.0m $\mathbf{x}$ + 3.0 $\mathbf{y}$; $\mathbf{d_3}$= 9.0m $\mathbf{x}$ + 4.0m$\mathbf{y}$

  You then found $\mathbf{d_T} = \mathbf{d_1} + \mathbf{d_2} + \mathbf{d_3}$

- The third set of instructions could be re-written as the sum of 4 two dimensional displacement vectors.

  $\mathbf{d_1}$= -49m $\mathbf{x}$ + 32m $\mathbf{y}$; $\mathbf{d_2}$= ______ $\mathbf{x}$ + ______ $\mathbf{y}$; $\mathbf{d_3}$= ______ $\mathbf{x}$ + ______ $\mathbf{y}$; $\mathbf{d_4}$= ______ $\mathbf{x}$ + ______ $\mathbf{y}$

  You then found $\mathbf{d_T} = \mathbf{d_1} + \mathbf{d_2} + \mathbf{d_3} + \mathbf{d_4}$

- The fourth set of instructions could be re-written as the sum of 3 two dimensional displacement vectors.

  $\mathbf{d_1}$= ______ $\mathbf{x}$ +______ $\mathbf{y}$; $\mathbf{d_2}$= ______ $\mathbf{x}$ + ______ $\mathbf{y}$; $\mathbf{d_3}$= ______ $\mathbf{x}$ + ______ $\mathbf{y}$;

  You then found $\mathbf{d_T} = \mathbf{d_1} + \mathbf{d_2} + \mathbf{d_3}$

The above examples were all given in terms of displacement vectors because that is easier for most people to visualize. The method, however, will work for any kind of two dimensional vectors, and can be easily generalized to 3-dimensions (or even 4 and 5… and n-dimensions).
In general what you did (whether you realize it or not) is add all of the x-components together (being careful to make right positive and left negative), and then add all of the y-components together (being careful to make up positive and down negative).

For the first example:

1. $\mathbf{d_1}$= -3.0m $\mathbf{x}$ + 4.0m $\mathbf{y}$; $\mathbf{d_2}$= -6.0m $\mathbf{x}$ + (-9.0m) $\mathbf{y}$; $\mathbf{d_3}$= 1.0m $\mathbf{x}$ + (-2.0m) $\mathbf{y}$

   $\mathbf{d_{1x}}$= -3.0m $\mathbf{x}$; $\mathbf{d_{1y}}$= 4.0m $\mathbf{y}$

   $\mathbf{d_{2x}}$= -6.0m $\mathbf{x}$; $\mathbf{d_{2y}}$= -9.0m $\mathbf{y}$

   $\mathbf{d_{3x}}$= 1.0m $\mathbf{x}$; $\mathbf{d_{3y}}$= -2.0m $\mathbf{y}$

Then $\mathbf{d_{Tx}} = \mathbf{d_{1x}} + \mathbf{d_{2x}} + \mathbf{d_{3x}}$ = (-3.0m + -6.0m + 1.0m) $\mathbf{x}$ = -8.0m $\mathbf{x}$ (8.0m left)

and $\mathbf{d_{Ty}} = \mathbf{d_{1y}} + \mathbf{d_{2y}} + \mathbf{d_{3y}}$ = (4.0m + -9.0m + -2.0m) $\mathbf{x}$ = -7.0m $\mathbf{y}$ (7.0m down)

So… $\mathbf{d_T}$ = -8.00m $\mathbf{x}$ + (-7.00m) $\mathbf{y}$

Again, we used displacement vectors for each of the above because they are the easiest to visualize, but the technique can be extended to any type of vector quantity: **velocity, acceleration, force, field, torque, momentum, impulse...**

Consider the example below.

Three forces are acting on a massive object. The object is a porcelain pig in a fish tank filled with mushroom broth. The forces are:

$\mathbf{F_1}$=12.0N $\mathbf{x}$ + (-18.0N)$\mathbf{y}$; $\mathbf{F_2}$= 9.0N$\mathbf{x}$ + 11.0N$\mathbf{y}$; $\mathbf{F_3}$=-16.0N$\mathbf{x}$ + (-2.0N)$\mathbf{y}$

Find the sum of these three forces:

$$\left.\begin{array}{llcl}
 & \mathbf{F_{1x}}= 12.0\text{N } \mathbf{x} & ; & \mathbf{F_{1y}}= -18.0\text{N } \mathbf{y} \\
+ & \mathbf{F_{2x}}= 9.0\text{N } \mathbf{x} & ; & \mathbf{F_{2y}}= 11.0\text{N } \mathbf{y} \\
+ & \mathbf{F_{3x}}= -16.0\text{N } \mathbf{x} & ; & \mathbf{F_{3y}}= -2.0\text{N } \mathbf{y} \\
\hline
= & \mathbf{F_{Tx}}= 5.0\text{N } \mathbf{x} & ; & \mathbf{F_{Ty}}= -9.0\text{N } \mathbf{y}
\end{array}\right\} \quad \mathbf{F_T}= 5.0\text{N } \mathbf{x} + (-9.0\text{N})\ \mathbf{y}$$

And that's it if the vectors are in component form!

Add the following vectors:

1. $\mathbf{v_1}$=6.0m/s $\mathbf{x}$ + 2.0m/s $\mathbf{y}$; $\mathbf{v_2}$=-1.0m/s $\mathbf{x}$ + 7.0m/s $\mathbf{y}$

2. $\mathbf{v_1}$=-1.0m/s $\mathbf{x}$ + 7.0m/s $\mathbf{y}$; $\mathbf{v_2}$=6.0m/s $\mathbf{x}$ + 2.0m/s $\mathbf{y}$

3. $\mathbf{v_1}$=6.0m/s $\mathbf{x}$ + 7.0m/s $\mathbf{y}$; $\mathbf{v_2}$=-1.0m/s $\mathbf{x}$ + 2.0m/s $\mathbf{y}$

4. $\mathbf{F_1}$=985N $\mathbf{x}$ + (-362N) $\mathbf{y}$; $\mathbf{F_2}$=-333N $\mathbf{x}$ + 68N $\mathbf{y}$; $\mathbf{F_3}$=128N $\mathbf{x}$ + 89N $\mathbf{y}$; $\mathbf{F_4}$=-435N $\mathbf{x}$ + (-699N) $\mathbf{y}$

Usually you will be given vector quantities in **standard form.** The reason is that you immediately know its magnitude and direction. If you are given a vector in component form you need to calculate the magnitude (using Pythagoras' Theorem) and you need to calculate the direction (using trigonometry). There is however a disadvantage to the standard form. The disadvantage comes when we attempt to add or subtract (or multiply) vector quantities.

Consider the following vector sum:

$\mathbf{d_1}$= 58.0m @ 63.5° above –x ; $\mathbf{d_2}$= 71.5m @ 13.2° below +x

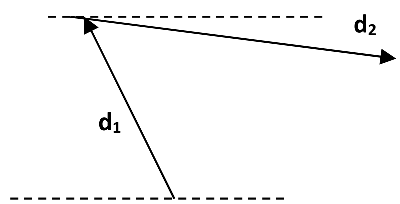
Find $\mathbf{d_3} = \mathbf{d_1} + \mathbf{d_2}$

Let's look at this carefully.

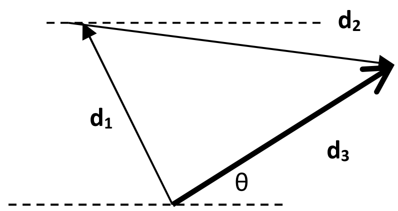
1. Draw each vector (to scale)
2. Add the vectors tail to head in the order given.
3. Draw the resultant vector, $\mathbf{d_3}$.

1.

2.



3.



Okay. So now what? It's a triangle, yes. But it is **NOT** a right triangle. So how can we find $||\mathbf{d_3}||$? How can we find θ?

Yes, you're right, we could use the sine law and the cosine law. But those are very cumbersome, and if we are not very careful, they give incorrect results. Any other thoughts?

Anything?
Hmmmmmm…

YES! OF COURSE!

Consider the following vector sum:

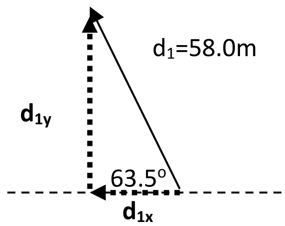
d_1= 58.0m @ 63.5° above –x ; **d_2**= 71.5m @ 13.2° below +x

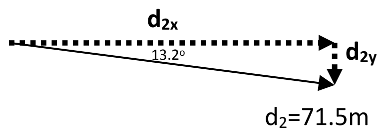
Find **$d_3 = d_1 + d_2$**

Let's look at this carefully.

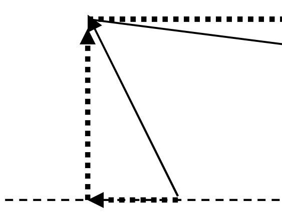
1. Draw each vector (to scale), and find the components
2. Add the vectors in the order given, include the components.
3. Remove the vectors and draw the resultant vector, **d_3**.
4. Rearrange the components (order of addition is not important!)
5. Solve the resulting right triangle.

1.

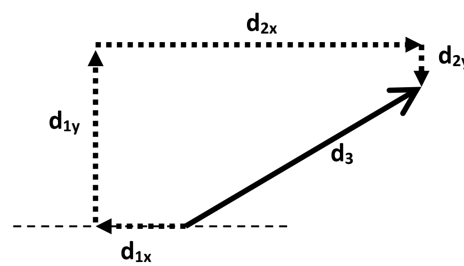




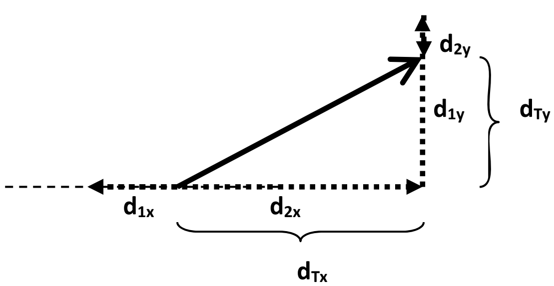
2.



3.



4.



(Can you see the right angle triangle?)

5. **d_{Tx}** = **d_{1x}** + **d_{2x}** = - 25.87947316m **x** + 69.61089156m **x** = 43.7314184m **x**
d_{Ty} = **d_{1y}** + **d_{2y}** = 51.90619297m **y** + (-16.32708721m) **y** = 35.57910576m **y**

$$d_T = \sqrt{d_{tx}^2 + d_{ty}^2} \qquad \theta = \tan^{-1}\frac{d_{ty}}{d_{tx}}$$

so.... **d_T**= ____