Significant Figures

In a measurement all of the measured digits INCLUDING THE ONE ESTIMATED DIGIT are considered to be significant. Digits (zeros) in a measured value that are there simply to show place value are not significant.

The rules for deciding the number of significant figures in a measurement are:

1. **Non-zero** digits are significant unless indicated otherwise.

2. Zeros **between** two significant digits are significant (for instance 205mL has three sig. figs.)

3. **Placeholder** zeros are not significant.

a. Zeros that lead a number, or come at the end of a number in order to show place value are not significant. For instance 0.003m has only one significant figure. The leading zeros are holding place value to show that the 3 represents 3 “one thousandths”, as opposed to 0.03m where the 3 represents 3 “one hundredths”.

b. Zeros at the end of a number that serve to hold place value are **not** significant. For instance 1200m contains two significant figures. The zeros at the end are to indicate that the 2 represents 2 “hundreds” and the 1 represents 1 “thousand”, as opposed to 120 where the 2 represents 2 “tens” and the 1 represents 1 “hundred”.

4. Zeros after the decimal point **and** after the first non-zero digit (i.e. “final zeros”) are significant.

For instance 3.200cm has four significant figures.

**Examples:**

A. 123km Non-zero digits are significant (Rule 1), so all digits are significant. 3significant figures.

B. 2001cm Non-zero digits are significant (Rule 1). The zeros in this measurement are between two non-zero, and thus significant figures, and so are also significant (Rule 2). 4 significant figures.

C. 0.0230s The two leading zeros are placeholders, and so are not significant (Rule 3a). The zero at the end is significant (Rule 4). 3 sig figs.

D. 1102200g Non-zero digits are significant (Rule 1). The zero between the 1 and 2 is significant (Rule 2). The final 2 zeros are placeholders, and so are not significant (Rule 3b). 5 sig figs

E. 20.0N Non zero digits are significant (Rule 1). The final zero is significant (Rule 4). The other zero is between two sig figs, and so is also significant (Rule 2). 3 sig figs

F. 1000000 Non zero digits are significant (Rule 1). All of the zeros are placeholders and so are not significant (Rule 3b). 1 sig fig.

Calculations with Significant Figures:

When we perform calculations we need to be very careful with our significant figures. Significant figures indicate the precision of a measurement. When we use these measurements to find other values we need to know how precise the calculated results are. Sig figs can be difference between whether the bridge stands or falls.

The following rules apply to calculations with significant figures:

1. **Addition and Subtraction**

Sums and differences of values are known only as well as the least PRECISE input value.

**Example 1.**

9400m + 822.2m = 10222.2m.

However this answer has an INCORRECT PRECISION.

The least precise input is 9400m, this is precise to 100m, thus the answer should be reported precise to 100m, or 10200m

So with proper significant figures:

9400m + 822.2m = 10200m

To see why, look at the operation:

* The number 9400m means that the measurement is between 9000m and 10000m. To the best estimate we have written 9400m. It is precise to 100m. The digit 4 is an ESTIMATE, the trailing zeros have no measurement value, they are place holders. The real value could be 9395m or 9428m or 9387.9m, ***we don’t know exactly!***
* The number 822.2m means that the measurement is between 822m and 823m and has been estimated to be 822.2m. It is precise to 0.1m. All of the digits in this number are significant.
* When we add (or subtract) we line up the decimal points like so…

 These zeros are INSIGNIFICANT. Their actual value could be anything!

 9 4 **0 0 . 0** m

 + 8 2 2 . 2 m

 1 0 2 2 2 . 2 m

These values were determined by adding INSIGNIFICANT zeros. They are

 not to be trusted!

 This is the last column containing ALL significant figures.

* So the answer should be reported as 10200m.

**Example 2.**

 78.6s + 59s

 7 8 . 6 s

 + 5 9 **. 0** s

 1 3 7 . 6 s

The final 6 is not significant, HOWEVER it is still used to help us round to find the final answer.

The final answer should be given as 138s

**Example 3.**

 418g + 950g – 99.576g

The least precise number here is 950g. It is precise to 10g. Thus our final answer must be given precise to 10g.

418g + 950g – 99.576g = 1268.43g

 = 1270g

2. **Multiplication and Division** (Including exponents and roots)

Products and quotients of values have the same number of significant figures as the input value with the least number of significant figures.

**Example 1:**

 29cm x 953cm = 27637 cm2

However this answer has an incorrect precision. The input with the least significant figures is 29cm, which has two sig figs. Thus the final answer should have two sig figs and be reported as 28000cm2.

The explanation for this rule is too complicated to go through here. So just trust me.

**Example 2.**

 $\frac{200m}{3.00s}$ = 66.6666666666666666666666666666666666666667m/s. Clearly this is TOO MANY digits.

200m has one sig fig, 3.00s has three sig figs, so the answer should be given with one sig fig.

$\frac{200m}{3.00s}$ = 70m/s

This rule also applies for trigonometric operations, square roots and other roots.

**Example 3.**

 If A=38.99m and B=11m, find $tan^{-1}\left(\frac{B}{A}\right)$ in degrees

 $tan^{-1}\left(\frac{11m}{38.99m}\right)$= 15.7550129o

 A has 4 sig figs, B has 2. The final answer should have 2. So…

$tan^{-1}\left(\frac{11m}{38.99m}\right)$= 16o

**Example 4.**

 6.6cm2=43.56cm2 but 6.6 has only 2 sig figs so the answer must be rounded to 2 sig figs:

 6.6cm2=44cm2

3. **Exact** physical and mathematical constants and defined conversion factors (π, 1kg = 1000g) integer coefficients (like the 2 in “C=2π*r*”), and the 10 in scientific notation (2.3x105m) have an **infinite** number of significant figures.

4. To avoid round-off errors, carry *at least* **2 insignificant** figures in intermediate calculations until the end of the **final** calculation. (Usually I keep all of the digits in my calculator)

5. Insignificant figures (excess digits) are rounded off in the **final** step of the calculation(s).