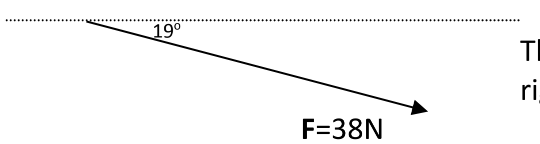
Converting between Standard and Component Form

It will be important for us to be able to work with vectors in both **standard** and **component** form. It is also important that we can convert a vector in standard form to a vector in component form, and to convert a vector in component form to a vector in standard form.

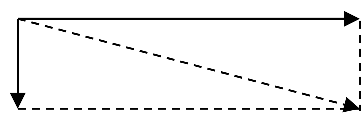
1. Standard to Component

Consider the following vector:

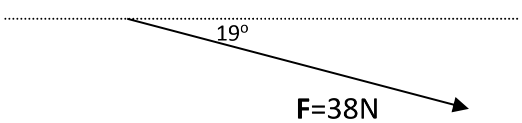
F = 38N [19° below +x]



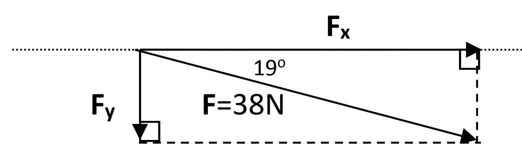
This vector points right AND down



But how do we find the x and y vectors (called **components**)? TRIGONOMETRY!



Now if we add a couple of lines we make 2 right triangles.



This vector can be described using its length, or magnitude and its direction, or angle.

We say: **F** = 38N [19° below +x] (Below because it is down, +x because it is to the right)

From trigonometry we can see: $\sin 19^\circ =$

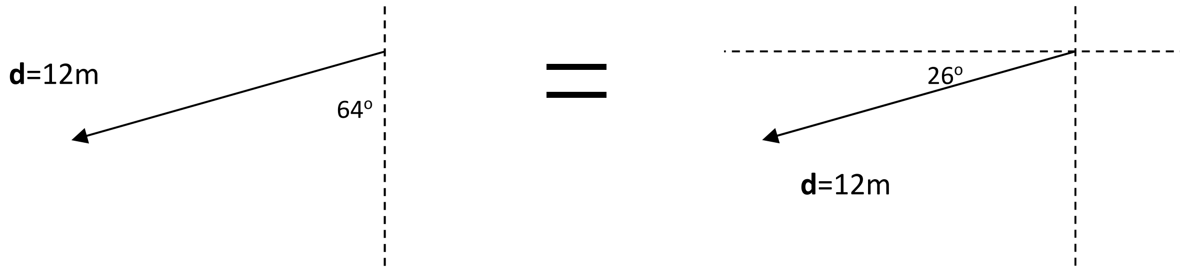
$\cos 19^\circ =$

$\tan 19^\circ =$

And so we can find the magnitudes of $\mathbf{F_x}$ and $\mathbf{F_y}$.

The direction is found just by looking at the diagram.

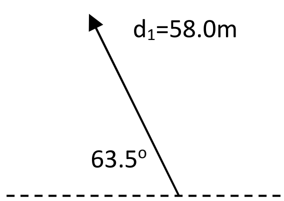
Example 2:

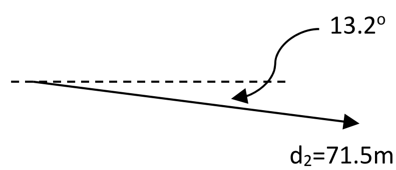


d = 12m [64° right of –y] = 12m [26° below –x] *

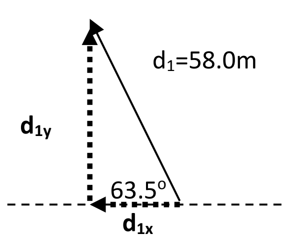
Sketch and find the x and y components of the above displacement vector

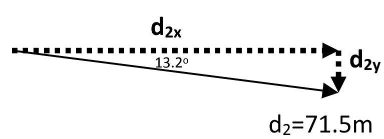
Lets illustrate with an example. Consider the following two vectors given in standard form.





$\mathbf{d_1}$ points left and up, $\mathbf{d_2}$ points right and down. The components of the vector simply tell us how much left or right and how much up or down. We can diagram this in a couple of ways.





This creates two right triangles! This is because BY DEFINITION ***the components of a vector are always perpendicular to each other.*** That's good because we know how to solve right angled triangles!

Consider $\mathbf{d_1}$ first. $\cos(63.5°)=\frac{d_{1x}}{d_1}$ $\Rightarrow$ $d_{1x} = d_1 \cos(63.5°) = 58.0\text{m} \cos(63.5°) = 25.87947316\text{m}$

Now we need to indicate that it points left. $\mathbf{d_{1x}}$ = - 25.87947316m **x**

$\sin(63.5^\circ) = \frac{d_{1y}}{d_1}$ ⇨ $d_{1y} = d_1 \sin(63.5^\circ) = 58.0\text{m} \sin(63.5^\circ) = 51.90619297\text{m}$

Now we need to indicate that it points up. $\mathbf{d_{1y}} = 51.90619297\text{m}\ \mathbf{y}$

We now know $\mathbf{d_1}$ in component form.

$$\mathbf{d_1} = -25.87947316\text{m}\ \mathbf{x} + 51.90619297\text{m}\ \mathbf{y}$$

Let's try $\mathbf{d_2}$ together: $\cos(13.2^\circ) = \frac{d_{2x}}{d_2}$ ⇨ $d_{2x} = d_2 \cos(13.2^\circ)$ = _______m cos(______°) = ____________________m

Now we need to indicate that it points _________. $\mathbf{d_{2x}}$ = ____________________

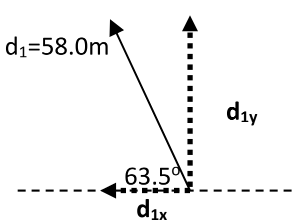
$\sin(13.2^\circ) =$ ——— ⇨ d_{2y} = ________________ = ________________ = _______________________

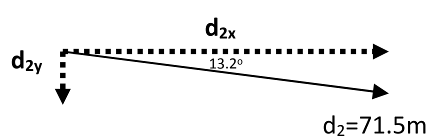
Now we need to indicate that it points _________. $\mathbf{d_{2y}}$ = ______________________

We now know $\mathbf{d_2}$ in component form.

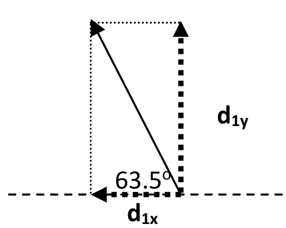
$\mathbf{d_2}$ = ____

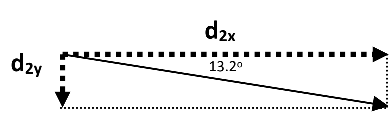
The second method is mathematically identical. The diagram just looks a little different.





Then form into rectangles with the original vector as the diagonal. This forms two right triangles.

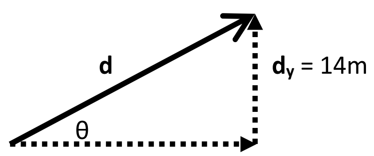




And now you can use the same trigonometry to find the components.

2. Component to Standard

This is fairly straight forward. All we need to be able to do is solve another right triangle.
Consider the following vector in component form:



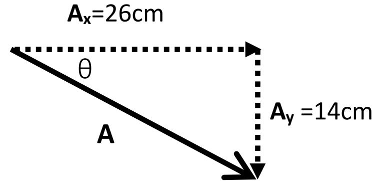
We can simply use Pythagoras' Theorem to find the magnitude, and the tangent ratio to find the direction.

$$d_T = \sqrt{{d_{tx}}^2 + {d_{ty}}^2} \qquad \theta = \tan^{-1}\frac{d_{ty}}{d_{tx}}$$

d_T= ____

Example: **A** = 26cm **x** + (-14cm) **y**

1. Sketch the vector:



2. Use Pythagoras' Theorem to find the magnitude:

$$A = \sqrt{{A_x}^2 + {A_y}^2} = \sqrt{26cm^2 + 14cm^2} = ...$$

3. Use the arctangent function to find the angle:

$$\theta = tan^{-1}\frac{A_y}{A_x} = tan^{-1}\frac{14cm}{26cm} = ...$$

4. Write the vector in standard notation:

A= _________cm [______° ______________________________]