Vectors in 2D: N2

As you know vectors have direction. In Physics 11 you will have seen vectors (forces, velocities, displacements, positions, accelerations) pointing left or right, up or down. Addition and subtraction of these vectors is simple; all you have to do is keep track of positive and negative to indicate directions.

In 1D objects move in straight lines. In 1D motion an object only has two choices for direction:

 If the motion is horizontal the object can move LEFT (-) or RIGHT (+)

 If the motion is vertical the object can move UP (+) or DOWN (-)

But what about a vector than points up AND left ( ) or left AND down ( ) or…? These are examples of vectors **in 2-Dimensions** and they are a *little* more complicated. The biggest difference is that in 2D there are more than just two directions; in fact there are INFINITE directions!

ET CETERA…

**IN 2-D THERE ARE *INFINITE* DIRECTIONS. OH MY!**

We need some way to describe EXACTLY what direction a given vector points:

Consider the following two vectors:

 5.0

 5.0

 $\rightharpoonaccent{A}$

 $ \rightharpoonaccent{B}$

Both vectors have a magnitude of 5.0, and both point LEFT and UP. It should be clear, however, that these vectors are not the same.

That is: $\rightharpoonaccent{A}\ne \rightharpoonaccent{B}$

The question is how to uniquely describe ANY vector in 2D. There are two common methods:

The Standard Form of Vectors:

In order to know a vector, you need to know its magnitude (how big?) and its direction. In two dimensions this can be done with a **magnitude** and a **single angle** (In 3-D space you need a magnitude and 2 angles). For example, the velocity of a frozen chicken shot from a frozen-chicken-cannon could be given as:

$\rightharpoonaccent{v}=144\frac{m}{s} [21.7^{o} above-x]$

This is known as standard form. The angle can be described in multiple ways. Consider the diagram below:

 **A B**

 2 3

 1 4

 8 6 5

 **C**

 7

 **D**

**This diagram is intended to indicate direction only. The lengths of the vectors are not meant to indicate relative magnitudes.**

Okay. We have four vectors, **A**, **B**, **C** and **D** and eight angles. Notice that the angles have direction; each angle is measured from either the x or y axis and ALL of the angles end on one of the vectors. The result is that each vector can be described by one of two angles.

* The direction of vector **A** can be described by angle 1 or angle 2.
* The direction of vector **B** can be described by angle 3 or angle 4.
* The direction of vector **C** can be described by angle 5 or angle 6.
* The direction of vector **D** can be described by angle 7 or angle 8.

Consider vector **A**. Assume the magnitude of **A,**  $\left‖A\right‖ $is 18m. Let’s further assume that θ1=39o , then we know θ2= \_\_\_\_\_\_\_o.

 The vector **A** can then be described in two ways:

 1. **A**= 18m [39o above –x]

 2. **A**= 18m [ 51 o left of +y]

Next consider vector **C**. Assume the magnitude of **C,**  $\left‖C\right‖ $is 120N. Let’s further assume that θ5=18o , then we know θ6= \_\_\_\_\_\_\_o.

 The vector **C** can then be described in two ways:

 1. **C**= 120N [18o below +x]

 2. **C**= 120N [ \_\_\_\_\_ o right of –y]

Next, let’s assume θ4=62o, θ7=27o, $\left‖B\right‖ $= 41m/s and $\left‖D\right‖$ = 7.6x105N/C. What are the two ways to describe **B** and **D**?

The vector **B** can then be described in two ways:

 1. **B** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 2. **B** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The vector **D** can then be described in two ways:

 1. **D** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 2. **D** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

*The standard convention is to measure the angle relative to the x-axis, using an angle LESS THAN 90o. So, unless otherwise stated, we will use angles 1, 4, 5 and 8.*

Consider the vectors below. Describe each vector in standard form (relative to the x-axis)

 S=25N

 36o

 67o 29o

 d=221km

 B=2.4x10-6T

 E=36000N/C

 F=38N

 141o

 16o

 73o

 Z=17m

 53o T=1500N 68o N=19m

 31o

 v=46m/s

$\rightharpoonaccent{S}$= 25N [36o above -x] ; $\rightharpoonaccent{B}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$\rightharpoonaccent{d}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{E}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$\rightharpoonaccent{Z}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{F}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$\rightharpoonaccent{T}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{N}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

$\rightharpoonaccent{v}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The Component Form of Vectors:

In order to know a vector you need to know its magnitude (how big?) and its direction. There are two basic ways to do this. You can know the **components** of the vector, or you can know the magnitude and angle(s) of the vector. In 2-D space any vector can be described with two perpendicular components (in 3-D space you need three perpendicular components.) *Most commonly* the components will be:

 Horizontal: This component tells you how much left or right. It is usually called the

x-component. Normally we will have positive values indicate right, and negative values indicate left.

 Vertical: This component tells you how much up or down. It is usually called the

y-component. Normally we will have positive values indicate up, and negative values indicate down.

Consider the following vector:

$\rightharpoonaccent{A}$= -6.00m $\hat{x}$ + 8.00m $\hat{y}$

This describes a SINGLE vector that is 6.00m left and 8.00m up. Below is a diagram of this vector:

 8.00m **A**

 θ

 6.00m

The vector itself is the heavy bold diagonal line\*. The dotted lines show the two components of the vector. The x-component of the vector is 6.00m left; the y component of the vector is 8.00m up. Most commonly the notation we will use is:

$\rightharpoonaccent{A}$**x**=-6.00m $\hat{x}$(negative to indicate left)

$\rightharpoonaccent{A}$**y**=8.00m $\hat{y}$(positive to indicate up)

Here is another example:

$\rightharpoonaccent{B}$= 1.4m/s $\hat{x}$ + (-0.98m/s) $\hat{y}$

This describes a SINGLE vector that is 1.4m/s to the right and 0.98m/s down. Below is a diagram of the vector.

1.4m/s

 $\rightharpoonaccent{B}$**x**= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

0.98m/s

 **B** $\rightharpoonaccent{B}$**y**= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\* Can you think of a way to find **A** in standard form?

Now Try these:

Consider the vector: $\rightharpoonaccent{G}$= 17N $\hat{x}$ + 31N $\hat{y}$**.**

1. In words this describes a vector that is \_\_\_\_\_\_\_\_\_ N to the \_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_N \_\_\_\_\_\_.

2. Sketch the vector.

3. $\rightharpoonaccent{G}$**x**= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{G}$**y**= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

Consider the vector: $\rightharpoonaccent{J}$= -420km $\hat{x }$+ (-136km)$\hat{ y}$.

4. In words this describes a vector that is \_\_\_\_\_\_\_\_\_ km to the \_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_km \_\_\_\_\_\_.

5. Sketch the vector.

6. $\rightharpoonaccent{J}$x= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ;$\rightharpoonaccent{ J}$y= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

Consider the vector: $\rightharpoonaccent{S}$= 0.998T $\hat{x }$+ (-0.21T) $\hat{y}$.

7. In words this describes a vector that is \_\_\_\_\_\_\_\_\_ T to the \_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_T \_\_\_\_\_\_.

8. Sketch the vector.

9. $\rightharpoonaccent{S}$x= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{S}$y= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

Consider the vector: $\rightharpoonaccent{E}$= -3.6x106N/C x + 5.9x106N/C y.

10. In words this describes a vector that is \_\_\_\_\_\_\_\_\_ N/C to the \_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_N/C \_\_\_\_\_\_.

11. Sketch the vector.

12. $\rightharpoonaccent{E}$x= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{E}$y= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ .

A displacement vector, $\rightharpoonaccent{Q}$, points 1.3m up and 2.2m left.

13. $\rightharpoonaccent{Q}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ $\hat{x}$ + \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$\hat{y}$

14. Sketch the vector.

15. $\rightharpoonaccent{Q}$x= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; $\rightharpoonaccent{Q}$y = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Consider the following vector: $\rightharpoonaccent{F}$x = -420N x ; $\rightharpoonaccent{F}$y = 930N y

16. In words this describes a vector that is \_\_\_\_\_\_\_\_\_ N to the \_\_\_\_\_\_\_, and \_\_\_\_\_\_\_\_\_\_\_N \_\_\_\_\_\_.

17. Sketch the vector.

18. $\rightharpoonaccent{F}$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

KEY KEY KEY

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In order to know a vector you need to know its magnitude (how big?) and its direction. In two dimensions any vector can be described with a **magnitude** and a **single angle** (In 3-D space you need a magnitude and 2 angles). The angle can be described in multiple ways. Consider the diagram below:

 **A B**

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Okay. We have four vectors, **A**, **B**, **C** and **D** and eight angles. Notice that the angles have direction; each angle is measured from either the x or y axis and ALL of the angles end on one of the vectors. The result is that each vector can be described by one of two angles.

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* The direction of vector **D** can be described by angle 7 or angle 8.

Consider vector **A**. Assume the magnitude of **A,**  $\left‖A\right‖ $is 18m. Let’s further assume that θ1=39o , then we know θ2= 51o.

 The vector **A** can then be described in two ways:

 1. **A**= 18m [39o above –x]

 2. **A**= 18m [51o left of +y]

Next consider vector **C**. Assume the magnitude of **C,**  $\left‖C\right‖ $is 120N. Let’s further assume that θ5=18o , then we know θ6= 72o.

 The vector **C** can then be described in two ways:

 1. **C**= 120N [18o below +x]

 2. **C**= 120N [72o right of –y]

Next, let’s assume θ4=62o, θ7=27o, $\left‖B\right‖ $= 41m/s and $\left‖D\right‖$ = 7.6x105N/C. What are the two ways to describe **B** and **D**?

The vector **B** can then be described in two ways:

 1. **B** = 41m/s [62o above +x]

 2. **B** = 41m/s [28o right of +y]

The vector **D** can then be described in two ways:

 1. **D** = 7.6x105N/C [63 below -x]

 2. **D** = 7.6x105N/C [27o left of –y]

*The standard convention is to measure the angle relative to the x-axis, using an angle LESS THAN 90o. So, unless otherwise stated, we will use angles 1, 4, 5 and 8.*

Consider the vectors below. Describe each vector in standard form (relative to the x-axis)

 S=25N

 36o

 67o 29o

 d=221km

 B=2.4x10-6T

 E=36000N/C

 F=38N

 141o 39o

 16o

 73o

 Z=17m

 53o T=1500N 68o N=19m

 31o

 v=46m/s

**S**= 25N [36o above -x] ; **B**= 2.4x10-6T [67o below –x]

**d**= 221km [29o below +x] ; **E**= 36000N/C [39o above +x]

**Z**= 17m [73o below +x] ; **F**= 38N [16o above –x]

**T**= 1500N [37o below +x] ; **N**= 19m [22o above +x]

**v**= 46m/s [59o below –x]

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y-component. Normally we will have positive values indicate up, and negative values indicate down.

Consider the following vector:

**A**= -6.00m **x** + 8.00m **y**

This describes a SINGLE vector that is 6.00m left and 8.00m up. Below is a diagram of this vector:

 8.00m **A**

 θ

 6.00m

The vector itself is the heavy bold diagonal line\*. The dotted lines show the two components of the vector. The x-component of the vector is 6.00m left; the y component of the vector is 8.00m up. Most commonly the notation we will use is:

 **Ax**=-6.00m **x** (negative to indicate left)

**Ay**=8.00m **y** (positive to indicate up)

Here is another example:

**B**= 1.4m/s **x** + (-0.98m/s) **y**

This describes a SINGLE vector that is 1.4m/s to the right and 0.98m/s down. Below is a diagram of the vector.

 **1.4m/s**

 **Bx**= 1.4m/s

0.98m/s

 **B By**= - 0.98m/s

\* Can you think of a way to find **A** in standard form? YES, I most certainly can.

Now Try these:

Consider the vector: **G**= 17N **x** + 31N **y.**

1. In words this describes a vector that is 17 N to the right, and 31N up.

2. Sketch the vector.

3. **Gx =** 17N$\hat{x}$; **Gy**= 31N $\hat{y}$

Consider the vector: **J**= -420km **x** + (-136km) **y.**

4. In words this describes a vector that is 420 km to the left, and 136 km down.

5. Sketch the vector.

6. **Jx**= -420km $\hat{x}$ ; **Jy**= -136km $\hat{y}$

Consider the vector: **S**= 0.998T **x** + (-0.21T) **y.**

7. In words this describes a vector that is 0.998T to the right, and 0.21Tdown.

8. Sketch the vector.

9. **Sx**= 0.998T $\hat{x}$ ; **Sy**= -0.21T $\hat{y}$

Consider the vector: **E**= -3.6x106N/C **x** + 5.9x106N/C **y.**

10. In words this describes a vector that is -3.6x106 N/C to the left, and 5.9x106 N/C up.

11. Sketch the vector.

12. **Ex**= -3.6x106 N/C $\hat{x}$ ; **Ey**= 5.9x106N/C $\hat{y}$

A displacement vector, **Q**, points 1.3m up and 2.2m left.

13. **Q**= -2.2m $\hat{x}$ + 1.3m $\hat{y}$

14. Sketch the vector.

15. **Qx**= -2.2m $\hat{x}$ ; **Qy** = 1.3m $\hat{y}$

Consider the following vector: **Fx** = -420N **x** ; **Fy** = 930N **y**

16. In words this describes a vector that is 420 N to the left, and 930N up.

17. Sketch the vector.

18. **F=** -420N $\hat{x }$+ 930N $\hat{y}$