Review of Right Angle Triangles

 You all should recall learning about right angled triangles in some math class at some dark point in your distant, or not-so-distant past. You likely have dusty, hazy recollections of Pythagoras’ Theorem and trigonometry rolling about in your head someplace (SOH CAH TOA, anyone?). You most likely thought that this knowledge was a waste of precious space in your brain, and that you would never need to use it again.

GREAT NEWS! You couldn’t have been more wrong! Right angle triangles are one of the foundations to understanding **vector algebra**. We will be using buckets full of trigonometry and Pythagoras’ theorem as we deal with vectors (displacement, velocity, force, field, acceleration, torque, flux and many more) in 2 and 3 dimensions.

The Pythagorean Theorem:

 Pythagoras was a brilliant (and eccentric) philosopher and mathematician who lived around 2500 years ago. His most famous mathematical theorem relates the sides of right angled triangles. The theorem states :

*“the area of the square on the hypotenuse (the side opposite the right angle)*

*is equal to the sum of the areas of the squares of the other two sides”*

You might recognize it better as

*“a2 + b2 = c2”*

Where a, b and c are the sides of a right angled triangle as shown below.

 *c*

 *a*

 *b*

Right Triangle Trigonometry:

 Right triangle trigonometry helps to relate the sides of right angles to the angles of the triangles. It does this through a series of trigonometric ratios with funny names. They are called Sine (sin), Cosine (cos), Secant (sec), Cosecant (csc), Tangent (tan) and Cotangent (cot). The most commonly used ratios (and the only ones we will use) are sin, cos and tan.

Let’s review the basic idea behind the three basic trigonometric ratios, **Sine (sin), Cosine (cos) and Tangent (tan)**, as they apply to right angled triangles.

Consider the following two triangles:

 F

 G

 φ

 E

 B

A

* φ

 C

Now; these triangles are not the same. The one on the left is larger. However they have something in common. What is it?

Right, they have the same angles.

 We could simply magnify the one on the right and it would match up with the one on left. Or we could shrink the one on the left to match it up with the one on the right. That makes these triangles very special friends. These are known as **similar triangles**. These triangles have all the same proportions; one is just a magnified/shrunken version of the other. Imagine a triangle drawn on the overhead projector and then the projector is rolled toward the screen or away. The overall SHAPE, the PROPORTIONS of the sides and the ANGLES of the projected triangle must match the one the teacher drew on the screen, even as the size changes.

All trigonometry does is define the PROPORTIONS or RATIOS of the sides, regardless of size. **ALL THAT MATTERS IS THE ANGLES!**

 θ

 F

 G

🞏 φ

 E

 θ B

A

 φ

 C

So for our triangles above the following ratios must be the same for both:

  

What trigonometry does is to define these ratios in terms of the angles in the **right triangle**. So in terms of the angle Φ:

 



In terms of the angle θ:

 



Examples and Practice

**The triangles in the following problems are all right triangles)**

1. Consider the following right triangle:

 9cm

θ

A. Which side is the hypotenuse?

B. How do you know? 15cm

C. Which side is opposite θ?

D. Which side is adjacent to θ?

E. Find the length of side J. J

2. Consider the following right triangle:

 24cm 10cm

A. Which side is the hypotenuse?

B. Which side is opposite the angle φ?

θ

φ

C. Which side is adjacent the angle φ?

D. Which side is opposite the angle θ? Q

E. Which side is adjacent the angle θ?

F. Find the length of side Q.

G. What is sinθ?

H. What is cosθ?

I. What is tanφ?

J. What is θ?

K. What is φ?

3. Consider the following right triangle:

M

A. What is the angle θ?

66o

B. What is cosθ?

C. What is sinθ?

D. What is cos66o?

123m

N

E. What is sin66o?

F. Find M.

θ

G. Find N.