

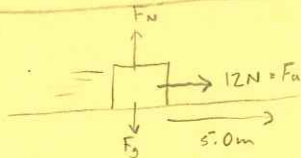
The Work Energy Theorem (Feat Momentum)

My usual preference is to use $\sum W_{nc} = \Delta E_k + \Delta E_p$ however for demonstrative purposes, I will switch back and forth between that formula and $\sum W = \Delta E_k$.

① $\sum W = \Delta E_k$;

$W_a + W_g + W_N = E_k - E_{k0}$

* these forces are perpendicular to \vec{d} so they do ZERO work, moving forward I will simply leave the perpendicular forces out!



$\rightarrow 12N(5.0m) + \frac{1}{2}(4.0kg)(2.0\frac{m}{s})^2 = \frac{1}{2}(4.0kg)v^2$

$v = 5.8 \frac{m}{s}$

$\vec{F}_{net} \cdot \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

OR

① $\sum W_{nc} = \Delta E_k + \Delta E_p$;

$W_a + W_N = E_k - E_{k0} + \Delta E_p$
(level ground, so no change is E_p)

$\rightarrow 12N(5.0m) + \frac{1}{2}(4.0kg)(2.0\frac{m}{s})^2 = \frac{1}{2}(4.0kg)(v^2)$

$v = 5.8 \frac{m}{s}$

$\vec{F}_{net} \cdot \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

② $\sum W = \Delta E_k$

$W_a = E_k - E_{k0}$

$\vec{F}_{net} \cdot \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$\rightarrow -3.0N(2.0m) + \frac{1}{2}(4kg)(2\frac{m}{s})^2 = \frac{1}{2}(4kg)v^2$

* \vec{F}_a is opposite direction to \vec{d} , so one +, one -.

$v = 1.0 \frac{m}{s}$

OR

$\sum W_{nc} = \Delta E_k + \Delta E_p$

$W_a = E_k - E_{k0}$

$\vec{F}_{net} \cdot \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$\rightarrow -3.0N(2.0m) + \frac{1}{2}(4kg)(2\frac{m}{s})^2 = \frac{1}{2}(4kg)v^2$

$v = 1.0 \frac{m}{s}$

$$\textcircled{3} \sum W = \Delta E_k$$

$$\vec{F}_{an} \cdot \vec{d} = E_k - E_{k0}$$

$$-N \cos 32^\circ (5.0m) = \frac{1}{2} 4kg v^2 - \frac{1}{2} 4kg (2m/s)^2$$

$$v = 5.4 \text{ m/s}$$

$$\textcircled{4} \sum W = \Delta E_k$$

$$W_a + W_g = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 \rightarrow 0$$

$$\vec{F}_{an} \cdot \vec{d} + \vec{F}_g \cdot \vec{d}_g = \frac{1}{2} m v^2$$

$$30N(6m) + (-6kg(9.8m/s^2))(2.0m) = \frac{1}{2} (6kg) v^2$$

* F_g is down, d_g is up, so...

$$\frac{180J - 117.6J}{3kg} = v^2$$

$$v = 4.6 \text{ m/s}$$

OR

$$\sum W_{nc} = \Delta E_k + \Delta E_p$$

$$W_a = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 + mgh - mgh_0 \rightarrow 0$$

$$\vec{F}_{an} \cdot \vec{d} = \frac{1}{2} 6kg v^2 + 6kg(9.8m/s^2)(2.0m)$$

$$v = 4.6 \text{ m/s}$$

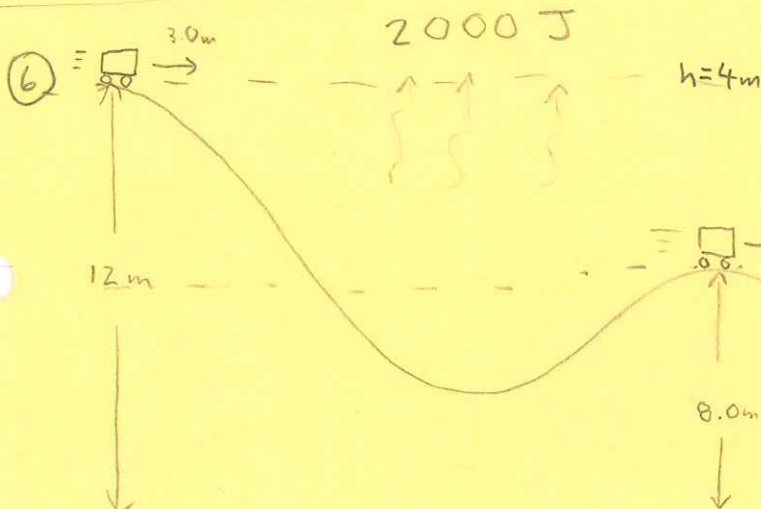
$$\textcircled{5} \sum W_{nc} = \Delta E_k + \Delta E_p$$

$$W_a + W_f = E_k - E_{k0} + E_p - E_{p0}$$

$$40N(6m) + -15N(6m) = \frac{1}{2} 6v^2 + 6(9.8)(2)$$

$$240J - 90J = 3kg v^2 + 117.6J$$

$$v = 3.3 \text{ m/s}$$



$$\sum W_{nc} = \Delta E_k + \Delta E_p$$

$$-2000J = E_k - E_{k0} + E_p - E_{p0}$$

$$-2000J = \frac{1}{2} 250v^2 - \frac{1}{2} 250(3)^2 - 250(9.8)4m$$

$$v = 8.4 \text{ m/s}$$

7) A. $W_{net} = \Delta E_k = E_k - E_{k0} = 220J - 880J = -660J$

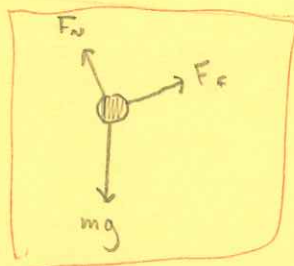
B. $W_{nc} = \Delta E_k + \Delta E_p = E_k - E_{k0} + E_p - E_{p0} = 220J - 880J + 640J - 450J = -470J$

C. $W_c = -\Delta E_p = -(E_p - E_{p0}) = -(640J - 450J) = -190J$

D. $\Delta E_k + \Delta E_p = -470J$

E. Heat and sound

8) A.



B. F_f and F_g

C. $\sum W = \Delta E_k = E_k - E_{k0} = \frac{1}{2}mv^2 = 11J$

D. $W_f = -7.00J$ (given)

E. $\sum W = W_f + W_g$

$W_g = \sum W - W_f$

$W_g = 11.16J - (-7.00J)$

$W_g = 18J$

F. $W_g = -\Delta E_{pg}$

$\Delta E_{pg} = -18.16J$

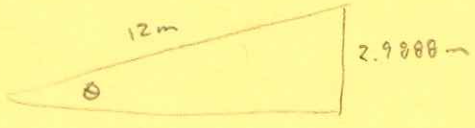
$\Delta E_{pg} = -18J$

G. $\Delta E_{pg} = mg \Delta h$

$\Delta h = -2.9888m$

$\Delta h = -3.0m$

H.

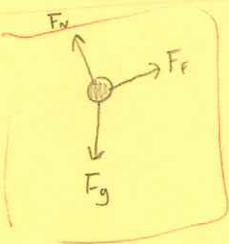


$\sin \theta = \frac{2.9888m}{12m}$

$\theta = 14^\circ$

9)

A.



B. F_f and F_g

C. $11J$

D. $W_f = -7.00J$ (HEAT)

E. $18J$

F. $-18J$

G. $-3.0m$

H. 14°

$$10. \quad W = \vec{F}_g \cdot \vec{d}$$

$$\left. \begin{aligned} v_0 &= 0 \text{ m/s} \\ \vec{a} &= -9.8 \text{ m/s}^2 \\ t &= 2.00 \text{ s} \end{aligned} \right\}$$

$$\vec{d} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{d} = 19.6 \text{ m down}$$

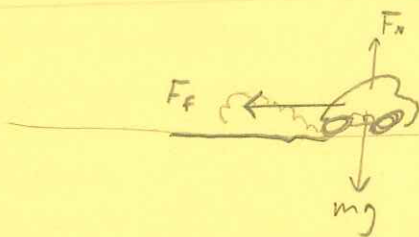
$$W = mg(19.6) = \boxed{23000 \text{ J}}$$

$$11. \quad \sum W_{nc} = \Delta E_k + \Delta E_p$$

$$W_f = \Delta E_k = E_k - E_{k0}$$

$$W_f = -\frac{1}{2} m v_0^2 = -\frac{1}{2} (1500 \text{ kg}) (20 \text{ m/s})^2$$

$$\boxed{W_f = -3.0 \times 10^5 \text{ J}}$$



$$B. \quad \sum W_{nc} = \Delta E_k + \Delta E_p$$

$$\vec{F}_f \cdot \vec{d} = E_k - E_{k0}$$

$$-\mu F_n d = -\frac{1}{2} m v^2$$

$$\rightarrow -\mu mg d = -\frac{1}{2} m v^2$$

$$\mu = \frac{\frac{1}{2} v^2}{g d} = \frac{\frac{1}{2} (20)^2}{9.8 (35)}$$

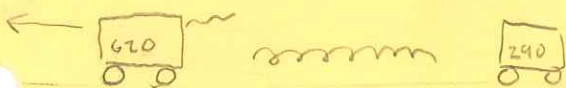
$$\boxed{\mu = 0.58}$$

$$12. \quad A. \quad \boxed{\theta_g = 47^\circ}$$

$$B. \quad \boxed{\theta_f = 180^\circ}$$

$$C. \quad \boxed{\theta_n = 90^\circ}$$

13.



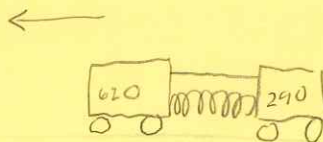
FINAL

$$U = 0$$

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

2.00 m/s



INITIAL

$$U_0 = U_s$$

$$K_0 = \frac{1}{2} (m_1 + m_2) v^2$$

$$\vec{p}_0 = (m_1 + m_2) \vec{v}_0$$

$$W_{nc} = \Delta K + \Delta U$$

$$0 = K - K_0 + U - U_0$$

$$U_0 = K - K_0$$

$$U_0 = \left[\frac{1}{2} (0.620)(2.75)^2 + \frac{1}{2} (0.290) v_2^2 \right] - \frac{1}{2} (0.620 + 0.290) (2.00)^2$$

there are 2 unknowns, I need a second eqn!

$$\Sigma \vec{p}$$

$$= \Sigma \vec{p}_0$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_0$$

$$(0.620)(-2.75) + 0.290 \vec{v}_2 = (0.620 + 0.290)(-2.00)$$

$$\vec{v}_2 = -0.396551725 \text{ m/s (sub into)}$$

$$U_0 = 0.547176724 \text{ J}$$

$$U_0 = 0.55 \text{ J}$$

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$$K = \frac{1}{2} m v^2$$

$$U_g = -\frac{GMm}{r}$$

It's strange, but for satellites in circular orbit

$$K = -\frac{1}{2} U$$



$$\Sigma \vec{F} = m \vec{a}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r} = mv^2$$

$$K = \frac{1}{2} \frac{GMm}{r}$$

COMPARE!

$$K = 2.5 \times 10^{11} \text{ J}$$

* This IS NOT A GENERAL FORMULA, but is true here

15



meteor

$$W_{nc} = \Delta K + \Delta U$$

$$-1.60 \times 10^9 \text{ J} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 + \frac{-GMm}{r_c} - \frac{-GMm}{r_c + h_0}$$

$$\frac{1}{2} m v^2 = -1.60 \times 10^9 \text{ J} + \frac{1}{2} m (2850 \text{ m/s})^2 + GMm \left(\frac{1}{r_c} - \frac{1}{r_c + 1234000} \right)$$

$$v = 5077.942629 \text{ m/s} \approx \boxed{5.08 \text{ km/s}}$$

16

INITIAL	FINAL
$K_0 = 6.4 \text{ J}$	$K = 7.7 \text{ J}$
$U_{g0} = 1.9 \text{ J}$	$U_g = 0.11 \text{ J}$
$U_{s0} = 3.3 \text{ J}$	$U_s = 2.0 \text{ J}$

$$W_{net} = \Delta K = K - K_0 = 7.7 \text{ J} - 6.4 \text{ J} = \boxed{1.3 \text{ J} = W_{net}}$$

$$W_g = -\Delta U_g = -(U_g - U_{g0}) = -(0.11 \text{ J} - 1.9 \text{ J}) = \boxed{1.8 \text{ J} = W_g}$$

$$W_s = -\Delta U_s = -(U_s - U_{s0}) = -(2.0 \text{ J} - 3.3 \text{ J}) = \boxed{1.3 \text{ J} = W_s}$$

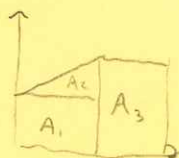
$$W_{nc} = \Delta K + \Delta U = K - K_0 + U - U_0 = K - K_0 + (U_g + U_s) - (U_{g0} + U_{s0})$$

$$W_{nc} = 7.7 \text{ J} - 6.4 \text{ J} + (0.11 \text{ J} + 2.0 \text{ J}) - (1.9 \text{ J} + 3.3 \text{ J}) = -1.79 \text{ J} = \boxed{-1.8 \text{ J} = W_{nc}}$$

17 $W = \text{Area}$

- a. $W = 80 \text{ N}(4 \text{ m}) = \boxed{320 \text{ J}}$
- b. $W = 80 \text{ N}(4 \text{ m}) = \boxed{320 \text{ J}}$
- c. $W = \frac{1}{2}(80)(8) = \boxed{320 \text{ J}}$
- d. $W = 80 \text{ N}(8 \text{ m}) + \frac{1}{2}(80 \text{ N})(8 \text{ m}) = \boxed{960 \text{ J}}$

18 $W = \text{Area} =$



$$W = A_1 + A_2 + A_3 = 10 \text{ N}(10 \text{ m}) + \frac{1}{2}(10 \text{ N})(10 \text{ m}) + 20 \text{ N}(6 \text{ m})$$

$$W = 270 \text{ J}$$

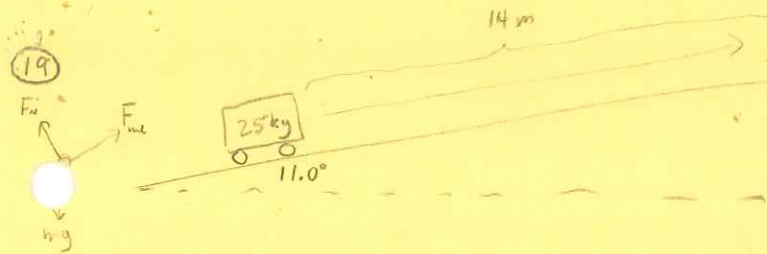
a. $\Sigma W = \Delta K = K - K_0$
 $270 \text{ J} = \frac{1}{2}(4.0 \text{ kg})v^2$
 $v = \boxed{12 \text{ m/s}}$

b. $W_{nc} = \Delta K + \Delta U$
 $270 \text{ J} = K - K_0 + U - U_0$
 $270 \text{ J} = \frac{1}{2}(4.0 \text{ kg})v^2 + mgh$
 $v = \boxed{8.4 \text{ m/s}}$

$\sin 12^\circ = \frac{h}{16}$

c. $W_{nc} = \Delta K + \Delta U$
 $270 \text{ J} = K - K_0 + U_g - U_{g0}$
 $270 \text{ J} = \frac{1}{2}(4)(v^2) - mgh_0$
 $v = \boxed{17 \text{ m/s}}$

$\sin 32^\circ = \frac{h_0}{16}$

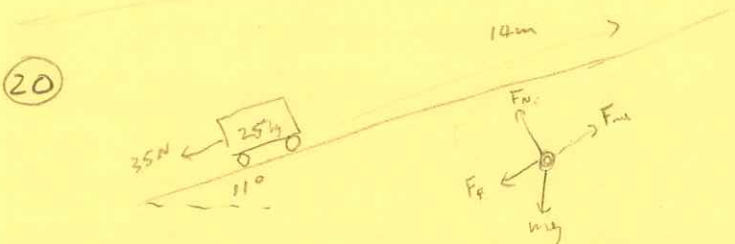


$$W_{nc} = \Delta K + \Delta U$$

$$W_{nc} = U_g - U_{g0} = mgh$$

$$W_{nc} = 25 \text{ kg} (9.8 \text{ m/s}^2) (14 \sin 11^\circ)$$

$$W_{nc} = 654 \text{ J}$$

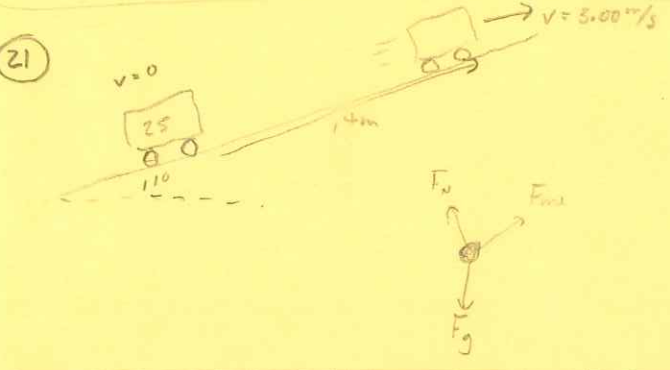


$$W_{nc} = \Delta K + \Delta U = U_g - U_{g0}$$

$$W_f + W_{nc} = mgh$$

$$-35(14) + W_{nc} = 25(9.8)(14 \sin 11^\circ)$$

$$W_{nc} = 1140 \text{ J}$$



$$W_{nc} = \Delta K + \Delta U$$

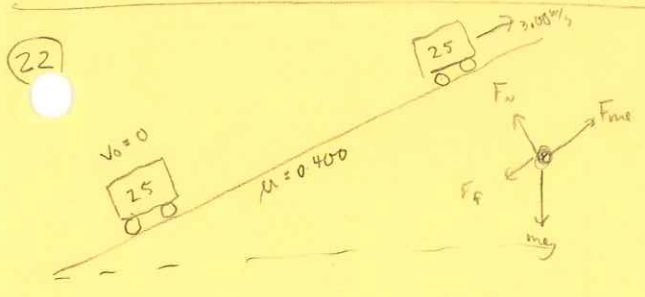
$$W_{gnc} = K - K_0 + U - U_0$$

$$W_{gnc} = \frac{1}{2}mv^2 + mgh$$

$$W_{gnc} = \frac{1}{2}(25)(3.00)^2 + 25(9.8)(14 \sin 11^\circ)$$

$$W_{gnc} = 766.97 \dots \text{ J}$$

$$W_{gnc} = 767 \text{ J}$$



$$\Sigma \vec{F}_x = 0$$

$$F_N = mg \cos 11^\circ$$

$$F_f = \mu F_N$$

$$W_{nc} = \Delta K + \Delta U$$

$$W_{nc} + W_f = K - K_0 + U - U_0$$

$$W_{nc} + (-\mu F_N)d = \frac{1}{2}mv^2 + mgh$$

$$W_{nc} = \frac{1}{2}(25)(3)^2 + 25(9.8)(14 \sin 11^\circ) + 0.400(25(9.8) \cos 11^\circ)14$$

$$W_{nc} = 2113.76 \approx 35 \text{ J}$$

$$W_{nc} = 2110 \text{ J}$$

$$\Sigma W = \Delta K = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(5)(2.36)^2 - \frac{1}{2}(5)(1.255)^2 = 9.9864375 \text{ J}$$

$$\Sigma W = 9.99 \text{ J}$$

$$\Sigma W = \Delta K = -220 \text{ J}$$

Energy Liberated as Heated Sound is ΣW_{nc} .

$$\Sigma W_{nc} = \Delta K + \Delta U = \Delta K + \Delta U_g + \Delta U_s = -333 \text{ J}$$

$$\Rightarrow 333 \text{ J lost}$$

$$(25) U_g = -\frac{GMm}{r} \Rightarrow U_g \propto \frac{1}{r}$$

$$\Rightarrow -880 \propto \frac{1}{D}$$

$$-\frac{880}{\frac{1}{2}} \propto \frac{1}{\frac{1}{2}D}$$

$$U_{g2} = -1760 \text{ J}$$

$$(26) E = K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$$

$$= \frac{1}{2} \frac{GMm}{r} + \frac{-GMm}{r}$$

$$= -\frac{1}{2} \frac{GMm}{r}$$

$$= -\frac{1}{2} \frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 (2.29 \times 10^{25} \text{ kg})(9999 \text{ kg})}{1.244505022 \times 10^8 \text{ m}}$$

$$= -6.14 \times 10^{10} \text{ J}$$

To Find K

$$\Sigma \vec{F} = m\vec{a}$$

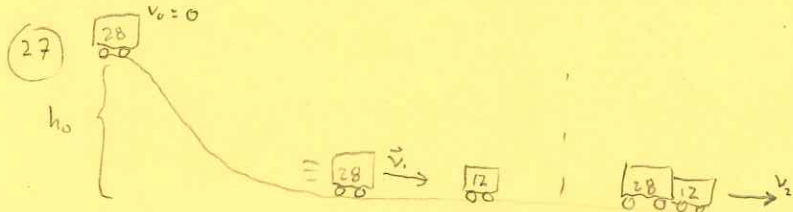
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r} = mv^2$$

To Find r

$$\frac{GMm}{r^2} = \frac{m4\pi^2 r}{T^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 1.244505022 \times 10^8 \text{ m}$$



$$w_{nc} = 0$$

$$E = E_0$$

$$\frac{1}{2}mv^2 = mgh_0$$

$$v_1 = \sqrt{2gh_0}$$

$$v_1 = 8.854377448 \text{ m/s}$$

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

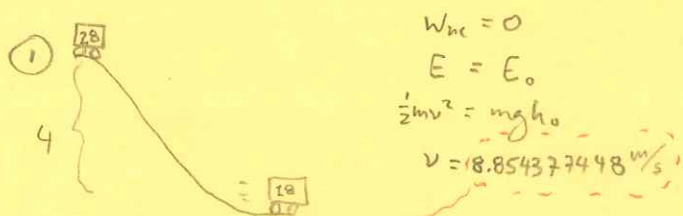
$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_2$$

$$\vec{v}_2 = 6.118064 \dots \text{ m/s right}$$

$$v = 6.2 \text{ m/s}$$

(28) This one has 3 parts! What a GREAT Q!

- 1) Find v of 28kg at bottom of hill (Energy)
- 2) Find \vec{v} of combined carts after collision (momentum)
- 3) Find d to stop (energy)

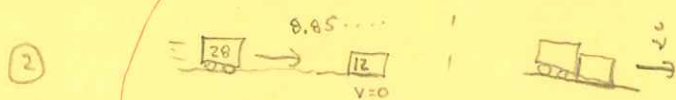


$$W_{nc} = 0$$

$$E = E_0$$

$$\frac{1}{2}mv^2 = mgh_0$$

$$v = 18.854377448 \text{ m/s}$$



$$\Sigma \vec{p}_0 = \Sigma \vec{p}$$

$$m_1 \vec{v}_{10} + m_2 \vec{v}_{20} = (m_1 + m_2) \vec{v}$$

$$\vec{v} = 6.198064214 \text{ m/s}$$

* F_f is only on the 12kg, but it must stop all 40kg!!



$$\Sigma W = \Delta K$$

$$W_f = K - K_0$$

$$\vec{F}_f \vec{d} = -\frac{1}{2}(m_1 + m_2) \vec{v}^2$$

$$-\mu m_2 g d = -\frac{1}{2}(m_1 + m_2) v^2$$

$$d = 10.05128205$$

$$d = 10.1 \text{ m}$$

29. a. $\sum W_{nc} = \Delta K + \Delta U$
 $U = U_0$
 $\frac{1}{2} kx^2 = m_1 g h_{10} + m_2 g h_{20}$

$x = 0.31 \text{ m}$

$x = 0.312084801 \text{ m}$

b. $\sum W_{nc} = \Delta K + \Delta U$
 $0 = K - K_0 + U - U_0$
 $K + U = U_0$
 $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} kx^2 = m_1 g h_{10} + m_2 g h_{20}$

$v_1 = 4.766885776 \text{ m/s}$

30. A. $W_{net} = \Delta K = 180 \text{ J} - 220 \text{ J} = -40 \text{ J}$
 B. $W_{nc} = \Delta K + \Delta U = 180 \text{ J} - 220 \text{ J} + (620 \text{ J} + 440 \text{ J}) - (380 \text{ J} + 40 \text{ J}) = 6.0 \times 10^2 \text{ J}$
 C. $W_c = -\Delta U$; $W_s = -\Delta U_s = -(440 \text{ J} - 40 \text{ J}) = -4.0 \times 10^2 \text{ J}$
 D. $W_g = -\Delta U_g = -(620 \text{ J} - 380 \text{ J}) = -240 \text{ J}$

31. $K_0 = 520 \text{ J}$
 $U_0 = 180 \text{ J}$
 $W_{net} = 180 \text{ J}$
 $W_{nc} = -260 \text{ J}$

A. $W_{net} = \Delta K$
 $180 \text{ J} = K - 520 \text{ J}$
 $K = 7.0 \times 10^2 \text{ J}$

B. $W_{nc} = \Delta K + \Delta U$
 $-260 = 180 \text{ J} + U - 180 \text{ J}$
 $U = -260 \text{ J}$

32. $\sum \vec{p}_i = \sum \vec{p}$
 $0 = m_1 \vec{v}_1 + m_2 \vec{v}_2$
 $0 = m_1 (-0.88 \text{ m/s}) + m_2 (3.52 \text{ m/s})$
 $m_1 = \frac{3.52}{0.88} m_2 = 4 m_2$

$W_{nc} = \Delta K + \Delta U$
 $0 = K - K_0 + U - U_0$
 $K = U_0$
 $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} kx^2$
 $4m_1^2 (0.88)^2 + m_2 (3.52)^2 = 999 (0.13)^2$

$m_2 = 1.090076188 \text{ kg}$
 $m_1 = 4.36 \dots \text{ kg}$

$m_1 = 4.4 \text{ kg}$
 $m_2 = 1.1 \text{ kg}$

$$33. \quad \vec{J} = \Delta \vec{p} = \vec{p} - \vec{p}_0 = \vec{p} + (-\vec{p}_0)$$

