The Work Energy Theorem:

My usual preference is to use $\sum W_{nc} = \Delta E_k + \Delta E_p$ however for demonstative purposes, I will switch back and forth between that formula and $\sum W = \Delta E_k$.

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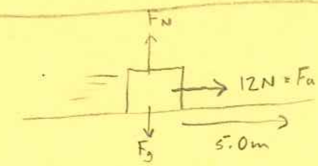
① $\sum W = \Delta E_k$;



$$W_a + \underbrace{\cancel{W_g}^{0} + \cancel{W_N}^{0}} = E_k - E_{k0}$$

* these forces are perpendicular to $\vec{d}$ so they do ZERO work, moving forward I will simply leave the perpendicular forces out!

$$\vec{F}_{a_{\parallel}} \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\rightarrow 12N(5.0m) + \frac{1}{2}(4.0kg)(2.0\,m/s)^2 = \frac{1}{2}(4.0kg)\,v^2$$

$$\boxed{v = 5.8\,m/s}$$

～～ OR ～～

① $\sum W_{nc} = \Delta E_k + \Delta E_p$;

$$W_a + \cancel{W_N}^{0} = E_k - E_{k0} + \cancel{\Delta E_p}^{0}$$

(level ground, so no CHANGE in E_{pg})

$$\vec{F}_{a_{\parallel}} \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\rightarrow 12N(5.0m) + \frac{1}{2}(4.0kg)(2.0\,m/s)^2 = \frac{1}{2}(4.0kg)(v^2)$$

$$\boxed{v = 5.8\,m/s}$$

---

② $\sum W = \Delta E_k$

$$W_a = E_k - E_{k0}$$

$$\vec{F}_{a_{\parallel}} \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\rightarrow -3.0N(2.0m) + \frac{1}{2}(4kg)(2\,m/s)^2 = \frac{1}{2}(4kg)\,v^2$$

* $\vec{F}_a$ is opposite direction to $\vec{d}$, so one +, one −.

$$\boxed{v = 1.0\,m/s}$$

～～ OR ～～

$\sum W_{nc} = \Delta E_k + \cancel{\Delta E_p}^{0}$

$$W_a = E_k - E_{k0}$$

$$\vec{F}_{a_{\parallel}} \vec{d} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\rightarrow -3.0N(2.0m) + \frac{1}{2}(4kg)(2\,m/s)^2 = \frac{1}{2}(4kg)\,v^2$$

$$\boxed{v = 1.0\,m/s}$$

③ $\sum W = \Delta E_k$

$\vec{F}_{a_\parallel}\vec{d} = E_k - E_{k0}$

$N\cos 32°(5.0m) = \frac{1}{2}4kg\,v^2 - \frac{1}{2}4kg(2\,m/s)^2$ → $\boxed{v = 5.4\,m/s}$

---

④ $\sum W = \Delta E_k$

$W_a + W_g = \frac{1}{2}mv^2 - \cancel{\frac{1}{2}mv_0^2}^{\,0}$

$\vec{F}_{a_\parallel}\vec{d} + \vec{F}_g\vec{d}_\parallel = \frac{1}{2}mv^2$

$30N(6m) + (-6kg(9.8\,N/kg)(2.0m)) = \frac{1}{2}(6kg)v^2$ → $\frac{180J - 117.6J}{3kg} = v^2$

$\boxed{v = 4.6\,m/s}$

* $\vec{F}_g$ is down, $\vec{d}_\parallel$ is up, so...

OR

$\sum W_{nc} = \Delta E_k + \Delta E_p$

$W_a = \frac{1}{2}mv^2 - \cancel{\frac{1}{2}mv_0^2}^{\,0} + mgh - \cancel{mgh_0}^{\,0}$

$\vec{F}_{a_\parallel}\vec{d} = \frac{1}{2}6kg\,v^2 + 6kg(9.8\,N/kg)(2.0m)$ → $\boxed{v = 4.6\,m/s}$

---

⑤ $\sum W_{nc} = \Delta E_k + \Delta E_p$

$W_a + W_f = E_k - \cancel{E_{k0}}^{\,0} + E_p - \cancel{E_{p0}}^{\,0}$

$40N(6m) + -15N(6m) = \frac{1}{2}6v^2 + 6(9.8)(2)$ → $240J - 90J = 3kg\,v^2 + 117.6J$

$\boxed{v = 3.3\,m/s}$

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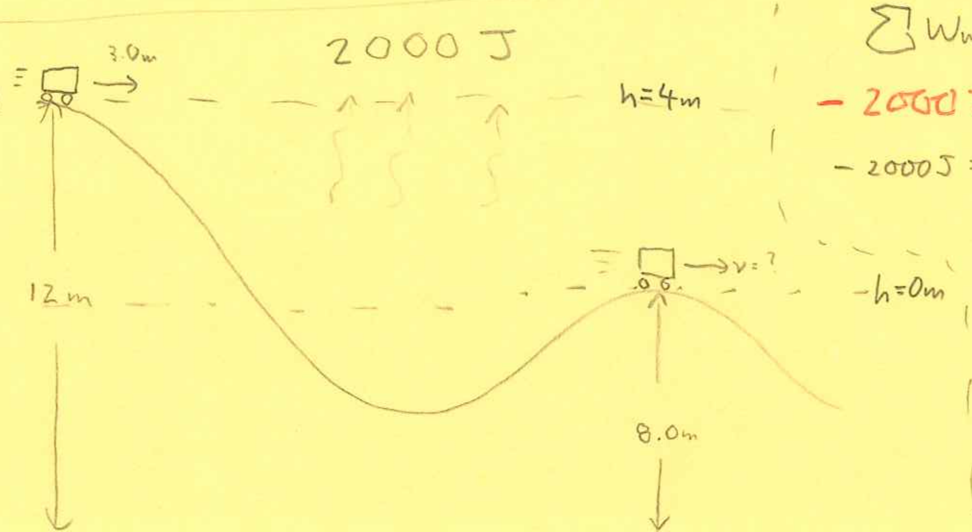
⑥



$\sum W_{nc} = \Delta E_k + \Delta E_p$

$-2000J = E_k - E_{k0} + \cancel{E_p}^{\,0} - E_{p0}$

$-2000J = \frac{1}{2}250v^2 - \frac{1}{2}250(3)^2 - 250(9.8)4m$

$\boxed{v = 8.4\,m/s}$

(7) A. $W_{net} = \Delta \mathcal{E}_k = \mathcal{E}_k - \mathcal{E}_{ko} = 220J - 880J = \boxed{-660\,J}$

B. $W_{nc} = \Delta \mathcal{E}_k + \Delta \mathcal{E}_p = \mathcal{E}_k - \mathcal{E}_{ko} + \mathcal{E}_p - \mathcal{E}_{po} = 220J - 880J + 640J - 450J = \boxed{-470J}$

C. $W_c = -\Delta \mathcal{E}_p = -(\mathcal{E}_p - \mathcal{E}_{po}) = -(640J - 450J) = \boxed{-190\,J}$

D. $\Delta \mathcal{E}_k + \Delta \mathcal{E}_p = \boxed{-470\,J}$

E. Heat and Sound

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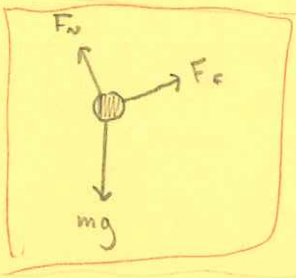
(8) A.



B. $\boxed{F_f \text{ and } F_g}$

C. $\sum W = \Delta \mathcal{E}_k = \mathcal{E}_k - \cancelto{0}{\mathcal{E}_{ko}} = \frac{1}{2}mv^2 = \boxed{11\,J}$

D. $W_f = \boxed{-7.00\,J}$ (Given)

E. $\sum W = W_f + W_g$

$W_g = \sum W - W_f$

$W_g = 11.16J - (-7.00J)$

$W_g = \boxed{18\,J}$

F. $W_g = -\Delta \mathcal{E}_{pg}$

$\Delta \mathcal{E}_{pg} = -18.16\,J$

$\Delta \mathcal{E}_{pg} = \boxed{-18\,J}$

G. $\Delta \mathcal{E}_{pg} = mg\Delta h$

$\Delta h = -2.9888\,m$

$\Delta h = \boxed{-3.0\,m}$

H.

$\sin\theta = \frac{2.9888m}{12m}$

$\boxed{\theta = 14°}$

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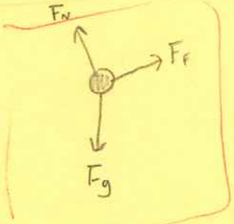
(9) A



B. $\boxed{F_f \text{ and } F_g}$

C. $\boxed{11\,J}$

D. $W_f = \boxed{-7.00J}$ (HEAT)

E. $\boxed{18\,J}$

F. $\boxed{-18J}$

G. $\boxed{-3.0m}$

H. $\boxed{14°}$

(10.) $W = \vec{F}_g \vec{d}$;

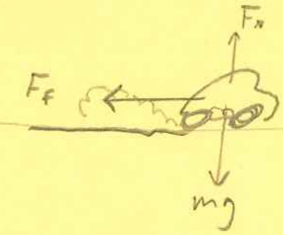
$v_0 = 0\ m/s$
$\vec{a} = -9.8\ m/s^2$
$t = 2.00\ s$

$\vec{d} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$\vec{d} = 19.6\,m$ down

$W = mg(-19.6) = \boxed{23\,000\ J}$

(11.) $\sum W_{nc} = \Delta E_k + \cancel{\Delta E_p}^{\,0}$



$W_f = \Delta E_k = \cancel{E_k}^{\,0} - E_{k0}$

$W_f = -\frac{1}{2} m v_0^2 = -\frac{1}{2}(1500\,kg)(20\,m/s)^2$

$\boxed{W_f = -3.0\times10^5\ J}$

B. $\sum W_{nc} = \Delta E_k + \cancel{\Delta E_p}^{\,0}$

$\vec{F}_f \vec{d} = \cancel{E_k}^{\,0} - E_{k0}$

$-\mu F_N d = -\frac{1}{2} m v^2$

$-\mu \cancel{m} g d = -\frac{1}{2}\cancel{m} v^2$

$\mu = \dfrac{\frac{1}{2}v^2}{g d} = \dfrac{\frac{1}{2}(20)^2}{9.8(35)}$

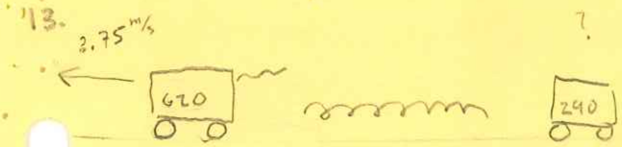
$\boxed{\mu = 0.58}$

(12.) A. $\boxed{\theta_g = 47^\circ}$

B. $\boxed{\theta_f = 180^\circ}$

C. $\boxed{\theta_N = 90^\circ}$

13.

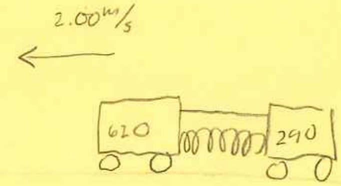


FINAL

$U = 0$

$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$

$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2$



INITIAL

$U_0 = U_s$

$K_0 = \frac{1}{2}(m_1 + m_2)v^2$

$\vec{p}_0 = (m_1 + m_2)\vec{v}_0$

$\cancel{W_{nc}}^{\,0} = \Delta K + \Delta U$

$0 = K - K_0 + \cancel{U}^{\,0} - U_0$

$U_0 = K - K_0$

$U_0 = \left[\frac{1}{2}(0.620)(2.75)^2 + \frac{1}{2}(0.290)v_2^2\right] - \frac{1}{2}(0.620+0.290)(2.00)^2$

there are 2 unknowns, I need a second eq'n!

$\Sigma\,\vec{p} = \Sigma\,\vec{p}_0$

$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_0$

$(0.620)(-2.75) + 0.290\,\vec{v}_2 = (0.620 + 0.290)(-2.00)$

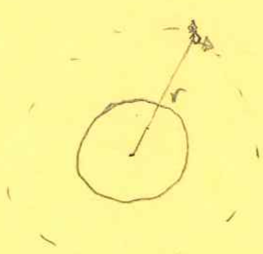
$\vec{v}_2 = -0.396551725\ m/s$ (sub into)

$U_0 = 0.547176724\ J$

$\boxed{U_0 = 0.55\ J}$

(14) $K = \frac{1}{2}mv^2$

$U_g = \frac{-GMm}{r}$



$\Sigma\vec{F} = m\vec{a}$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$\frac{GMm}{r} = mv^2$

$K = \frac{1}{2}\frac{GMm}{r}$

COMPARE!

It's strange, but for satellites in circular orbit

$K = -\frac{1}{2}U$ *

$\boxed{K = 2.5\times10^{11}\ J}$

* This IS NOT A GENERAL FORMULA, but is true here

15


meteor

$$W_{nc} = \Delta K + \Delta U$$

$$-1.60\times10^{10}\,J = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + \frac{-GMm}{r_e} - \frac{-GMm}{r_e+h_0}$$


Earth

$$\tfrac{1}{2}mv^2 = -1.60\times10^{10}\,J + \tfrac{1}{2}m(2850\,m/s)^2 + GMm\left(\frac{1}{r_e} - \frac{1}{r_e+1234000}\right)$$

$$v = 5077.942629\,m/s \approx \boxed{5.08\,km/s}$$

16

INITIAL	FINAL
$K_0 = 6.4J$	$K = 7.7J$
$U_{g0} = 1.9J$	$U_g = 0.11J$
$U_{s0} = 3.3J$	$U_s = 2.0J$

$$W_{net} = \Delta K = K - K_0 = 7.7J - 6.4J = \boxed{1.3J = W_{net}}$$

$$W_g = -\Delta U_g = -(U_g - U_{g0}) = -(0.11J - 1.9J) = \boxed{1.8J = W_g}$$

$$W_s = -\Delta U_s = -(U_s - U_{s0}) = -(2.0J - 3.3J) = \boxed{1.3J = W_s}$$

$$W_{nc} = \Delta K + \Delta U = K - K_0 + U - U_0 = K - K_0 + (U_g + U_s) - (U_{g0} + U_{s0})$$

$$W_{nc} = 7.7J - 6.4J + (0.11J + 2.0J) - (1.9J + 3.3J) = -1.79J = \boxed{-1.8J = W_{nc}}$$

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17 W = Area

a. $W = 80N(4m) = \boxed{320J}$

b. $W = 80N(4m) = \boxed{320J}$

c. $W = \tfrac{1}{2}(80)(8) = \boxed{320J}$

d. $W = 80N(8m) + \tfrac{1}{2}(80N)(8m) = \boxed{960J}$

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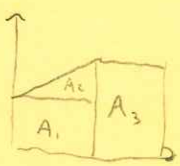
18 W = Area =



$$W = A_1 + A_2 + A_3 = 10N(10m) + \tfrac{1}{2}(10N)(10m) + 20N(6m)$$

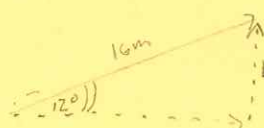
$$W = 270J$$

a. $\Sigma W = \Delta K = K - \cancel{K_0}^{\,0}$

$270J = \tfrac{1}{2}(4.0kg)v^2$

$\boxed{v = 12\,m/s}$

b.



$W_{nc} = \Delta K + \Delta U$

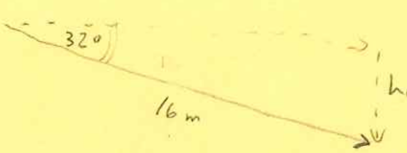
$270J = K - \cancel{K_0} + U - \cancel{U_0}^{\,0}$

$270J = \tfrac{1}{2}4.0kg\,v^2 + mgh$

$\sin 12° = \frac{h}{16}$

$\boxed{v = 8.4\,m/s}$

c.



$W_{nc} = \Delta K + \Delta U$

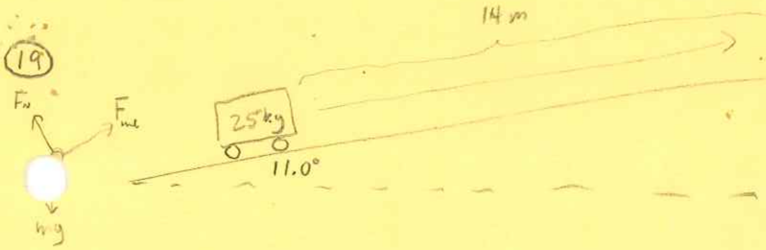
$270J = K - \cancel{K_0}^{\,0} + \cancel{U_g}^{\,0} - U_{g0}$

$270J = \tfrac{1}{2}4(v^2) - mgh_0$

$\sin 32° = \frac{h_0}{16}$

$\boxed{v = 17\,m/s}$

(19)



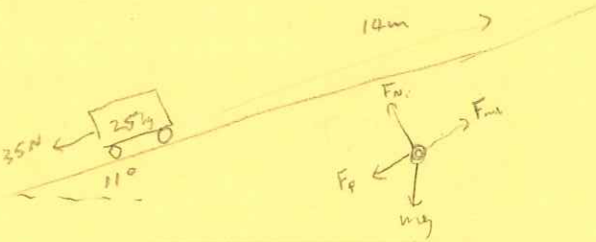
$W_{nc} = \cancel{\Delta K}^{0} + \Delta U$

$W_{me} = U_g - \cancel{U_{g0}}^{0} = mgh$

$W_{me} = 25\,kg\,(9.8\,N/kg)(14 \sin 11°)$

$\boxed{W_{me} = 654\ J}$

(20)



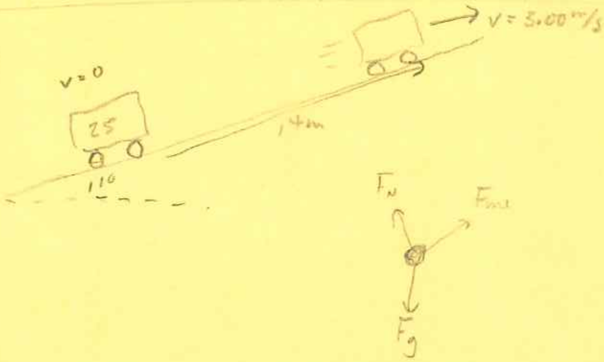
$W_{nc} = \cancel{\Delta K}^{0} + \Delta U = U_g - \cancel{U_{g0}}^{0}$

$W_f + W_{me} = mgh$

$-35(14) + W_{me} = 25(9.8)(14 \sin 11°)$

$\boxed{W_{me} = 1140\ J}$

(21)



$W_{nc} = \Delta K + \Delta U$

$W_{you} = K - \cancel{K_0}^{0} + U - \cancel{U_0}^{0}$

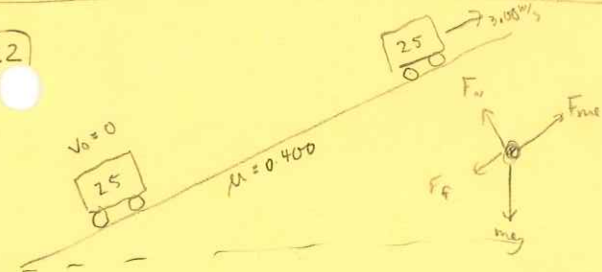
$W_{you} = \frac{1}{2}mv^2 + mgh$

$W_{you} = \frac{1}{2}(25)(3.00)^2 + 25(9.8)14 \sin 11°$

$W_{you} = 766.97 \ldots J$

$\boxed{W_{you} = 767\ J}$

(22)



$\Sigma \vec{F}_y = 0$

$F_N = mg\cos 11°$

$F_f = \mu F_N$

$W_{nc} = \Delta K + \Delta U$

$W_{me} + W_f = K - \cancel{K_0}^{0} + U - \cancel{U_0}^{0}$

$W_{me} + (-\mu F_N)d = \frac{1}{2}mv^2 + mgh$

$W_{me} = \frac{1}{2}(25)(3)^2 + 25(9.8)14 \sin 11° + 0.400(25(9.8)\cos 11°)14$

$W_{me} = 2113.76735\ J$

$\boxed{W_{me} = 2110\ J}$

(23) $\Sigma W = \Delta K = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}5(2.36)^2 - \frac{1}{2}(5)(1.255)^2 = 9.9864375\ J$

$\boxed{\Sigma W = 9.99\ J}$

(24) $\Sigma W = \Delta K = \boxed{-220\ J}$

→ Energy Liberated as Heat and Sound is ΣW_{nc}

$\Sigma W_{nc} = \Delta K + \Delta U = \Delta K + \Delta U_g + \Delta U_s = -333\ J$

$\Rightarrow \boxed{333\ J \text{ lost}}$

(25) $U_g = -\frac{GMm}{r} \Rightarrow U_g \propto \frac{1}{r}$

$\Rightarrow -880 \propto \frac{1}{D}$

$\frac{-880}{\frac{1}{2}} \propto \frac{1}{\frac{1}{2}D}$

$\boxed{U_{g2} = -1760\ J}$

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(26) $E = K + U = \frac{1}{2}mv^2 + \left(\frac{-GMm}{r}\right)$

$= \frac{1}{2}\frac{GMm}{r} + \frac{-GMm}{r}$

$= -\frac{1}{2}\frac{GMm}{r}$

$= -\frac{1}{2}\frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}(2.29\times10^{25}kg)(9999kg)}{1.244505022\times10^{8}m}$

$\boxed{= -6.14\times10^{10}\ J}$

To Find K

$\Sigma\vec{F} = m\vec{a}$

$\frac{GMm}{r^2} = \frac{mv^2}{r}$

$\frac{GMm}{r} = mv^2$

To Find r

$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}4\pi^2 r}{T^2}$

$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = 1.244505022\times10^{8}m$

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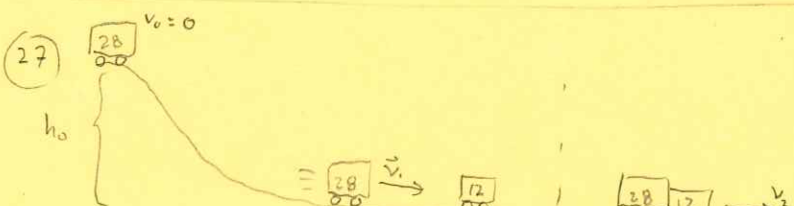
(27)



$W_{nc} = 0$

$E = E_0$

$\frac{1}{2}mv_1^2 = mgh_0$

$v_1 = \sqrt{2gh_0}$

$v_1 = 8.854377448\ m/s$

$\Sigma\vec{p}_0 \stackrel{?}{=} \Sigma\vec{p}$

$m_1\vec{v}_1 + \cancel{m_2\vec{v}_{20}}^{0} = (m_1+m_2)\vec{v}_2$

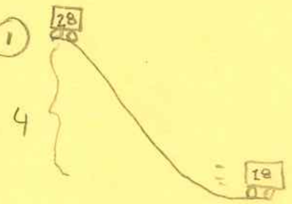
$\vec{v}_2 = 6.198064\ldots\ m/s$ right

$\boxed{v = 6.2\ m/s}$

(28) This one has 3 parts! What a GREAT Q!

(1) Find v of 28kg at bottom of hill (Energy)
(2) Find $\vec{v}$ of combined carts after collision (momentum)
(3) Find $\vec{d}$ to stop (energy)

(1)



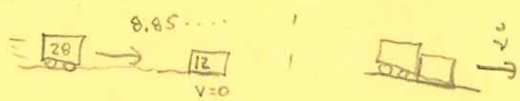
$W_{nc} = 0$

$E = E_0$

$\frac{1}{2}mv^2 = mgh_0$

$v = 8.854377448 \, m/s$

(2)



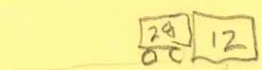
$\Sigma \vec{p}_0 = \Sigma \vec{p}$

$m_1\vec{v}_{10} + m_2\vec{v}_{20} = (m_1+m_2)\vec{v}$

$\vec{v} = 6.198064214 \, m/s$

* F_f is only on the 12kg, but it must stop all 40kg!!

(3)



$\Sigma W = \Delta K$

$W_f = \cancel{K} - K_0$

$\vec{F}_f \vec{d} = -\frac{1}{2}(m_1+m_2)v^2$

$-\mu m_2 g d = -\frac{1}{2}(m_1+m_2)v^2$

$d = 10.05128205$

$\boxed{d = 10.1 \, m}$

(29) a. $\cancel{\Sigma W_{nc}}^{0} = \cancel{\Delta K}^{0} + \Delta U$

$U = U_0$

$\frac{1}{2}kx^2 = m_1 g h_{10} + m_2 g h_{20}$ → $\boxed{x = 0.31 m}$

$x = 0.312084801 m$

b. $\cancel{\Sigma W_{nc}}^{0} = \Delta K + \Delta U$

$0 = K - \cancel{K_0}^{0} + U - U_0$

$K + U = U_0$

$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}kx^2 = m_1 g h_{10} + m_2 g h_{20}$

$\boxed{v_1 = 4.766885776 \, m/s}$

30. A. $W_{net} = \Delta K = 180J - 220J = -40J$

B. $W_{nc} = \Delta K + \Delta U = 180J - 220J + (620J + 440J) - (380J + 40J) = 6.0 \times 10^2 J$

C. $W_c = -\Delta U$; $W_s = -\Delta U_s = -(440J - 40J) = -4.0 \times 10^2 J$

D. $W_g = -\Delta U_g = -(620J - 380J) = -240J$

31. $K_0 = 520J$

$U_0 = 180J$

$W_{net} = 180J$

$W_{nc} = -260J$

A. $W_{net} = \Delta K$

$180J = K - 520J$

$K = 7.0 \times 10^2 J$

B. $W_{nc} = \Delta K + \Delta U$

$-260 = 180J + U - 180J$

$U = -260J$

32. $\cancel{\Sigma \vec{p}_0}^{0} = \Sigma \vec{p}$

$0 = m_1\vec{v}_1 + m_2\vec{v}_2$

$0 = m_1(-0.88 m/s) + m_2(3.52 m/s)$

$m_1 = \frac{3.52}{0.88} m_2 = 4 m_2$

$\cancel{W_{nc}}^{0} = \Delta K + \Delta U$

$0 = K - \cancel{K_0}^{0} + \cancel{U}^{0} - U_0$

$K = U_0$

$\cancel{\frac{1}{2}}m_1 v_1^2 + \cancel{\frac{1}{2}}m_2 v_2^2 = \cancel{\frac{1}{2}}kx^2$

$4m_2(0.88)^2 + m_2(3.52)^2 = 999(0.13)^2$

$m_2 = 1.090076188 kg$

$m_1 = 4.36 \ldots kg$

$\boxed{m_1 = 4.4 kg \quad m_2 = 1.1 kg}$

33. $\vec{J} = \Delta\vec{p} = \vec{p} - \vec{p}_0 = \vec{p} + (-\vec{p}_0)$