Base 10 and Scientific Notation:

**Base 10 (Decimal) Numbers:**

We (by “we” I mean “human beings”) use the decimal number system, which is based on powers of ten. Consider the number 62150. This number is sixty-two thousand one hundred fifty. This number is sixty thousand + two thousand + one hundred + fifty. This number is six groups of ten thousand + two groups of one thousand + one group of one hundred + five groups of ten + zero groups of one. This number is 6x104 + 2x103 + 1x102 + 5x101 + 0x100 . Each number in any decimal number represents some power of ten. Notice that even though there are zero groups of one we still have to include the ones when we write the number. If we left off the ones (because, after all, there are none of them) the number would be 6215, which is very different. We need the zero to serve as a ‘place holder’ to show that the 5 represents groups of ten and not groups of one.

What power of ten each digit represents is determined by its position relative to the *decimal point*. The first digit to the left of the decimal point tells you how many groups of one there are, the next digit to the left tells you how many groups of ten, the next to left tells you how many hundreds, one more left thousands, then ten thousands, hundred thousands, millions etc. Digits to the right of a decimal tell you many tenths, hundredths, thousandths etc.

Look at this number: 4008.2045

Here is what it means in our decimal system.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **4** | **0** | **0** | **8** | **2** | **0** | **4** | **5** |
| Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths | Ten-Thosandths |
| 103 | 102 | 101 | 100 | 10-1 | 10-2 | 10-3 | 10-4 |

**Scientific Notation:**

Scientific notation is a method of writing numbers in terms of powers of 10. This is convenient as we use a base 10 (decimal) number system. Scientific notation is particularly useful when we need to write very large numbers, very small numbers or if we need to make the precision of a number clear.

Consider the following number: 3450000000. This number means 3x1010 + 4x109 + 5x108 + 0x107 + 0x106 + 0x105 + 0x104 + 0x103 + 0x102 + 0x101 + 0x100. All of those zeros can be left off in the exponent form, so 3450000000=3x1010 + 4x109 + 5x108. You have likely noticed that this is not actually saving us any time. But lets look at this more closely.

Confirm that the following is true: 4 = 0.4x10

5 = 0.05x102

Right? Why yes it is, but so what? Well, check this out.

3450000000 = 3x1010+4x109+5x108 = 3x1010+0.4x10x109+0.05x102x108 = 3x1010+0.4x1010+0.05x1010

Let’s factor that last part: (3+0.4+0.05)x1010 = 3.45x1010

One more example, but let’s skip some steps:

50900 = 5x104 + 9x102 = 5x104 + 0.09x104 = (5 + 0.09)x104 = 5.09x104

That was a lot of work, but we don’t ever have to do it again. As long you understand the concept, all you need to do is figure out the largest power of ten of any digit in the number, move the decimal next to that digit and rewrite the number.

 The result is a lead number (N), between 1 and 10, 1 ≤ N < 10, (often just called the lead for short) multiplied by some power of ten.

**Examples:**

 a. 604500 Here the 6 represents the largest power of ten. It represents 6 x 105. So from what we figured out in the rather lengthy explanation above we can rewrite this as 6.045 x 105

 b. 7770000000000000000 This is annoying. You have to count out to figure out how big this number is. When we do we see that the largest power of 10 is 1017. So 7.77x1017

 c. 0.009032 Same rule applies. Which digit represents the LARGEST power of ten? That’s right the 9. What power is it? It represents 9 one thousandths, or 9x10-3, so we can rewrite this number as 9.032x10-3

 d. 124x104 This number is NOT in scientific notation. The lead is not between 1 and 10, so what shall we do? Deal with the lead first. 124=1.24x102, thus, 124x104 = 1.24x102x104=1.24x106

**Calculations Using Scientific Notation**

Scientific notation can also be very helpful when performing calculations or making estimates. If I asked you to calculate 314000 x 2000 you would likely go straight for your calculator. Well, cut it out. No calculators allowed! Let’s put these into scientific notation and see how it helps.

314000 x 2000 = (3.14 x 105) x (2 x 103), now the order in which we multiply doesn’t matter (right?). so

this is the same as 3.14 x 2 x 105 x 103 = 6.28 x 108 . Yay.

How about this one: 7x102 x 3x109? Okay, this is the same as 7x3 x 102x109 = 21x1011. Terrific. But not perfect. What’s wrong? The lead is not between one and ten. 21x1011 = 2.1x10x1011 = 2.1x1012

We can do the same with division. Try this without a calculator: 8.4x1013 ÷ 4x1010

This is the same as $\frac{8.4×10^{13}}{4×10^{10}}$ = $\frac{8.4}{4} × \frac{10^{13}}{10^{10}}$ = 2.1 x 103 . Did you get it?

Of course sometimes we won’t be able to do the calculation in your head, but we can still us these ideas to help ESTIMATE the answer. This can be very useful, particularly on multiple choice tests.

Examples: 2.334x106(1.86x107).

I personally can’t multiply 2.334 by 1.86 in my head, if you can, congratulations! What I can do is see that 2.334 is a little bigger than 2, and 1.86 is a little smaller than 2, so I can approximate 2.334 x 1.86 as 2 x 2 = 4. What I can do in my head is 106 x 107 = 1013. So 2.334x106(1.86x107) ≈ 4x1013 . If you check on your calculator 2.334x106(1.86x107) = 4.34124x1013, so 4x1013 is a pretty good estimate. Especially if the question on a multiple choice test looks like:

**1. Find the product of 2.334x106 and 1.86x107:**

**a. 3.99x102 b. 7.28x1011 c. 4.45x109 d. 4.34x1013**

Notice that you could choose the best answer by only looking at the power of ten.

Try these without a calculator:

2. Find the product of 4.1x1015 and 2.02x106:

 a. 8.3x1021 b. 3.4x1018 c. 9.9x1034 d. 5.1x10-4 e. 3.4x1011

3. Find the product of 6.99x105 and 3.89x104.

 a. 2.72x109 b. 1.13x1010 c. 8.84x106 d. 1.13x109 e. 2.72x1010

4. Find the quotient: 8.6x1012 divided by 2.0x109.

 a. 4.3x1021 b. 4.3x103 d. 3.2x103 e. 7.6x103 f. 1.7x1021

**Scientific Notation and your Calculator**

 Any scientific (or graphing) calculator will have a scientific notation mode. That means the calculator will put numbers into scientific notation for you. While that is very nice, you will be expected to do it yourself without a calculator. Take some time right now to find where the scientific notation mode is on your calculator. Go ahead, I’ll wait.

 Even in scientific notation mode, you still must know how to properly input the numbers into your calculator. Again your calculator is your friend and it provides you with a short cut. In order to enter the number **6.23x1018** into your calculator do the following:

 1. Punch in 6.23

 2. Find the key on your calculator that says one of the following: “ EXP ” “ EE ” “ x10*x* ”

 3. Punch that key

4. Punch in 18

There done. Now try the following using your calculator, the correct answer is given so you can check your work.

 1. 1.226x104(8.55x109) Answer: 1.04823x1014

 2. 5.5x10-9(6.11x1018) Answer: 3.3605x1010

 3. 3.7x105 + 9.9x10-15 Answer: 3.7x105

 4. 6.98x10-5 ÷ 1.0x10-9 Answer: 6.98x104

 5. 4πx10-7 ÷ 3.2x1022 Answer: 3.926990817x10-29

 6. $\frac{55.2×10^{4}}{2π}$ Answer: 8.78535286x104