Scientific Notation:

**Exponents:**

Exponents are a sort of mathematical shorthand. If we have a number, any number, or a variable (which is like a number in disguise), multiplied by ITSELF repeatedly we can use exponents to save time (and graphite).

Ex. 2(2)(2)(2), which is 2 multiplied by itself four times, can be written 24.

 a(a)(a)(a)(a)(a)(a)(a) can be written a8.

 5(5)(5)(5)(5)(5)(5)(5)(5)(5)(5)(5)(5)(5) can be written 514.

You can imagine what writing out 688 would be like without exponents.

**Powers of 10**

All of this applies to any number, but we are interested in powers of ten. So consider the following powers of 10. You can check all of these on your calculator if you wish.

 104 = 10x10x10x10 = 10 000

 103 = 10x10x10 = 1000

 102 = 10x10 = 100

 101 = 10 = 10

 100 = 1 = 1

 10-1 = $\frac{1}{10}$ = 0.1

 10-2 = $\frac{1}{10×10}$ = 0.01

 10-3 = $\frac{1}{10×10×10}$ = 0.001

 10-4 = $\frac{1}{10×10×10×10}$ = 0.0001

Notice that if you count the number of zeroes in the decimal form it tells you the exponent. This is because our whole number system is based on tens. We use a BASE 10, or DECIMAL number system This can also be thought of as how many places we need to move the decimal point.

**Base 10 (Decimal) Numbers:**

Consider the number **62150**. This number is **sixty-two thousand one hundred fifty**.You may recall doing something like the following in elementary school.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ten thousands | thousands | hundreds | tens | ones |
| 6 | 2 | 1 | 5 | 0 |

Let’s do the same thing in a slightly more mathy way:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ten thousands | thousands | hundreds | tens | ones |
| 6 | 2 | 1 | 5 | 0 |
| 104 | 103 | 102 | 101 | 100 |

* This number is **sixty thousand + two thousand + one hundred + fifty**.
* This number is **six groups of ten thousand + two groups of one thousand + one group of one hundred + five groups of ten**.
* This number is **6x104 + 2x103 + 1x102 + 5x101 + 0x100** .

Each number in any decimal number represents some **power of ten**. The zero at any position in the number is very important. If we left off the ones (because, after all, there are none of them) the number would be 6215, which is very different. We need the zero to serve as a ***‘place holder’*** to show that the 5 represents groups of ten and not groups of one.

What power of ten each digit represents is determined by its **position relative to the *decimal point***. The first digit to the left of the decimal point tells you how many groups of one there are, the next digit to the left tells you how many groups of ten, the next to left tells you how many hundreds, one more left thousands, then ten thousands, hundred thousands, millions etc. Digits to the right of a decimal tell you many tenths, hundredths, thousandths etc.

We can use this to write some numbers in a simple short-hand.

**Scientific Notation**

Primarily, scientific notation is simply a shorter way to write numbers that have a lot of zeroes at the beginning or the end. The number is written as a lead number 1 and 10 (1$\leq $ N $<$ 10) multiplied by the appropriate power of ten.

 **Decimal (Standard) Form Expanded Form Scientific Notation**

 500 5 x 100 5 x 102

 200000 2 x 100000 2 x 105

 0.004 4 x 0.001 4 x 10-3

 0.0000000007 7 x 0.0000000001 7 x 10-10

This is particularly useful for very large and very small numbers.

Some numbers are a bit more complicated.

 Use your calculator to confirm the following:

 12 = 1.2 x 10

 560 = 5.6 x 100 = 5.6 x 102

 0.037 = 3.7 x 0.01 = 3.7 x 10-2

 238400 = 2.384 x 100000 = 2.348 x 105

0.00099 = 9.9 x 0.0001 = 9.9x10-4

For each of these examples if you look at the **largest power of ten** in the number you can figure out what power of ten you need to use in scientific notation.

In the number 560 the “5” is in the hundreds (102) position, the 6 is in the tens (101) position, the 0 is in the ones (100) position. So the largest power of 10 in this number is 102. We can thus write 520 as 5.2 x 102.

In the number 0.037 the 3 is in the hundredths (10-2) position and the 7 is in the thousandths (10-3) position. So the largest power of ten is 10-2 (-2 > -3). We can thus write 0.037 as 3.7 x 10-2

But as a shortcut all we need to do is count how far we need to move the decimal point. Look at the last two examples:

 238400: How far do we need to move the decimal to get it between the 2 and 3?

 238400

 5 4 3 2 1

Five places. That means we can write this as 2.384x10**5**

 0.00099: How far do we need to move the decimal to get it between the first and second 9?

So…

 **Decimal (Standard) Form Expanded Form Scientific Notation**

 560 5.6 x 100 5.6x102

 204000 2.04 x 100000 2.04x104

 0.00000000074 7.4 x 0.0000000001 7.4x10-10

Look at this number: 4008.2045

Here is what it means in our decimal system.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **4** | **0** | **0** | **8** | **decimal point** | **2** | **0** | **4** | **5** |
| Thousands | Hundreds | Tens | Ones | Tenths | Hundredths | Thousandths | Ten-Thosandths |
| 103 | 102 | 101 | 100 | 10-1 | 10-2 | 10-3 | 10-4 |

What we can do in the decimal system is re-write the number by figuring the HIGHEST power of ten in the number, then writing the number as a decimal between 1 and 10 (1$\leq $ N $<$ 10), multiplied by that power of ten.

The highest power or 10 in 4008.2045 is 103. We can re-write this number as: 4.0082045 x 103.

How about 250 000 000? The highest power of ten is 109. We can re-write this as 2.5x109.

How about 0.0000457? The highest power of ten is 10-5 . We can re-write this as 4.57X10-5

**More Examples:**

604500 Here the 6 represents the largest power of ten. It represents 6 x 105. So we can rewrite this as

6.045 x 105

7770000000000000000 This is annoying. You have to count out to figure out how big this number is. When we do we see that the largest power of 10 is 1017. So 7.77x1017

0.009032 Same rule applies. Which digit represents the LARGEST power of ten? That’s right the 9. What power is it? It represents 9 one thousandths, or 9x10-3, so we can rewrite this number as 9.032x10-3

124x104 This number is **NOT** in scientific notation. The lead is not between 1 and 10, so what shall we do? Deal with the lead first. 124=1.24x102, thus, 124x104 = 1.24x102x104=1.24x106

**Practice:**

Write the following in scientific notation:

A. 20 000 B. 0.005 C. 0.00008 D. 900000000000 E. 0.224 F. 135 G. 300250000

H. 0.00087 I. 1500 J. 363000 K. 0.000000000044 L. 85 M. 0.0077 N. 13 000 000

Write the following in scientific notation:

A. 12x103 B. 130x104 C. 0.06x106 D. 0.15x10-5 E. 255x10-10

Write the following in standard form:

A. 2.5x103 B. 2.5x10-3 C. 6x106 D. 4.0079 x 103 E. 8.87x10-7

**Scientific Notation and your Calculator**

 Any scientific (or graphing) calculator will have a scientific notation mode. That means the calculator will put numbers into scientific notation for you. While that is very nice, you will be expected to do it yourself without a calculator. Take some time right now to find where the scientific notation mode is on your calculator. You may need to search through some settings menu somewhere. Look for a key that says FSE or perhaps SCI. Often it is above a key, so you will need to use the second function key, 2ndF. Go ahead, I’ll wait.

 Even in scientific notation mode, you still must know how to properly input the numbers into your calculator. Again your calculator is your friend and it provides you with a short cut. In order to enter the number 6.23x1018 into your calculator do the following:

 1. Punch in 6.23

 2. Find the key on your calculator that says one of the following: “EXP” “EE” “x10*x*” “x10*y*”

 3. Punch that key

4. Punch in 18

There done. Now try the following using your calculator, the correct answer is given so you can check your work.

 1. 1.226x104(8.55x109) Answer: 1.04823x1014

 2. 5.5x10-9(6.11x1018) Answer: 3.3605x1010

 3. 3.7x105 + 9.9x10-15 Answer: 3.7x105

 4. 6.98x10-5 ÷ 1.0x10-9 Answer: 6.98x104

 5. 4πx10-7 ÷ 3.2x1022 Answer: 3.926990817x10-29

 6. $\frac{55.2×10^{4}}{2π}$ Answer: 8.78535286x104